



EE 338 - Digital Signal Processing

Filter Design Assignment

Mantri Krishna Sri Ipsit
180070032
Autumn 2020

Abstract

This report is made as a part of the Evaluation Component - 4 of the Digital Signal Processing happening in Autumn 2020 at IIT Bombay. This report summarizes the filter design techniques for designing a bandpass and a bandstop filter of Butterworth and Chebyshev nature.

Acknowledgements

I would like to thank and I am grateful to Professor V.M.Gadre for introducing and teaching me this fundamental and important course of Digital Signal Processing. The tasks that were a part of this course helped a lot in learning and understanding different aspects in this field, which even the ongoing pandemic couldn't stop. I would also like to thank my group, Mr. Shlok Vaibhav Singh and Mr. Amol Girish Shah for helping me and collaborating with me in various aspects of this course.

Author

Mantri Krishna Sri Ipsit
Group - 1, Filter Number - 44
Roll No: 180070032
Junior Undergraduate
Department of Electrical Engineering
Indian Institute of Technology Bombay

Instructor

Prof. Vikram.M.Gadre
Department of Electrical Engineering
Indian Institute of Technology Bombay

Peer Reviewer

Shlok Vaibhav Singh
Roll No: 18D070064
Junior Undergraduate
Department of Electrical Engineering
Indian Institute of Technology Bombay

Contents

1	Infinite Impulse Response Bandpass Filter	1
1.1	Unnormalized Specifications	1
1.2	Normalized Specifications	1
1.3	Analog Filter Specifications using Bilinear Transform	2
1.4	Analog Frequency Transformation	2
1.5	Equivalent Analog Lowpass Filter Specifications	3
1.6	Analog Lowpass Magnitude Response	3
1.7	Analog Bandpass Transfer Function	7
1.8	Discrete-Time Bandpass Transfer Function	8
2	Finite Impulse Response Bandpass Filter	11
2.1	Implementation Technique	11
2.2	Kaiser Window Parameters	12
2.3	Discrete Time FIR Filter Transfer Function	12
2.4	Magnitude and Phase Response	13
2.5	Comparison with the IIR Filter	14
3	Infinite Impulse Response Bandstop Filter	15
3.1	Unnormalized Specifications	15
3.2	Normalized Specifications	15
3.3	Analog Filter Specifications using Bilinear Transform	16
3.4	Analog Frequency Transformation	17
3.5	Equivalent Analog Lowpass Filter Specifications	17
3.6	Analog Lowpass Magnitude Response	18
3.7	Analog Bandstop Transfer Function	20
3.8	Discrete-Time Bandstop Transfer Function	22
4	Finite Impulse Response Bandstop Filter	24
4.1	Implementation Technique	24
4.2	Kaiser Window Parameters	25
4.3	Discrete Time FIR Filter Transfer Function	25
4.4	Magnitude and Phase Response	26
4.5	Comparison with the IIR Filter	27

5	Elliptical Bandpass filter design	28
5.1	Unnormalized Specifications	28
5.2	Normalized Specifications	28
5.3	Analog Filter Specifications using Bilinear Transform	29
5.4	Analog Frequency Transformation	29
5.5	Equivalent Analog Lowpass Filter Specifications	30
5.6	Analog Lowpass Magnitude Response	30
5.6.1	Jacobian Elliptic Functions	31
5.6.2	Elliptic Filter Parameters	32
5.7	Analog Bandpass Transfer Function	34
5.8	Discrete-Time Bandpass Transfer Function	35
6	Elliptic Bandstop filter design	37
6.1	Unnormalized Specifications	37
6.2	Normalized Specifications	37
6.3	Analog Filter Specifications using Bilinear Transform	38
6.4	Analog Frequency Transformation	39
6.5	Equivalent Analog Lowpass Filter Specifications	39
6.6	Analog Lowpass Magnitude Response	40
6.6.1	Jacobian Elliptic Functions	40
6.6.2	Elliptic Filter Parameters	41
6.7	Analog Bandstop Transfer Function	43
6.8	Discrete-Time Bandstop Transfer Function	44
7	Conclusion	46
8	Peer Review	47
8.1	Review Received	47
8.2	Review Given	47
9	Appendix	48

1 Infinite Impulse Response Bandpass Filter

The filter number $m = 44$. Hence,

$$q(m) = \lfloor 0.1m \rfloor = 4$$

$$r(m) = m - 10q(m) = 44 - 40 = 4$$

$$BL(m) = 25 + 1.7q(m) + 6.1r(m) = 55.8 \text{ kHz}$$

$$BH(m) = BL(m) + 20 = 75.8 \text{ kHz}$$

1.1 Unnormalized Specifications

- Lower Passband Edge = 55.8 kHz
- Upper passband Edge = 75.8 kHz
- Transition band width = 4 kHz
- Passband tolerance = 0.15
- Stopband tolerance = 0.15
- Passband nature = monotonic
- Stopband nature = monotonic
- Sampling frequency = 330 kHz

1.2 Normalized Specifications

The sampling angular frequency $2\pi f_s$ should map to 2π after normalization. Hence the transformation used is

$$\omega = 2\pi \frac{f_s}{f}$$

The normalized specifications are as follows (following the notations used in the lecture)

- $\omega_{p1} = 2\pi \frac{55.8}{330} = 0.3382 \pi$
- $\omega_{p2} = 2\pi \frac{75.8}{330} = 0.4594 \pi$

- Transition bandwidth = $2\pi \frac{4}{330} = 0.0242 \pi$
- $\omega_{s1} = \omega_{p1} - 0.0242 \pi = 0.314 \pi$
- $\omega_{s2} = \omega_{p2} + 0.0242 \pi = 0.4836 \pi$
- Passband and Stopband tolerance = 0.15
- Passband and Stopband nature = monotonic

1.3 Analog Filter Specifications using Bilinear Transform

We use the following transformation to move from the z domain to the s domain.

$$s = \frac{1 - z^{-1}}{1 + z^{-1}}$$

If we put $z = j\omega$ and $s = j\Omega$, we get

$$\Omega = \tan \frac{\omega}{2}$$

The specifications now are

- Passband and Stopband nature = monotonic
- $\Omega_{s1} = 0.5375$
- $\Omega_{s2} = 0.9498$
- $\Omega_{p1} = 0.5875$
- $\Omega_{p2} = 0.8799$
- $\delta_1 = \delta_2 = 0.15$

1.4 Analog Frequency Transformation

Here we use the **Bandpass to Lowpass** analog frequency transformation, which is as follows:

$$\Omega_L = \frac{\Omega^2 - \Omega_0^2}{B\Omega}$$

where

$$\Omega_0 = \sqrt{\Omega_{p1} \Omega_{p2}} = 0.7189$$

and

$$B = \Omega_{p2} - \Omega_{p1} = 0.2924$$

Using the transformation mentioned above, we now have

- Passband and Stopband nature = monotonic
- $\Omega_{Ls1} = -1.4501$
- $\Omega_{Ls2} = 1.3874$
- $\Omega_{Lp1} = -1$
- $\Omega_{Lp2} = +1$
- $\delta_1 = \delta_2 = 0.15$

1.5 Equivalent Analog Lowpass Filter Specifications

Using the more stringent specifications from those obtained in section 1.4, we have

- Passband and Stopband nature = monotonic
- $\Omega_{Ls} = \min\{\Omega_{Ls2}, |\Omega_{Ls1}|\} = 1.3874$
- $\Omega_{Lp} = +1$
- $\delta_1 = \delta_2 = 0.15$
- $D_1 = \frac{1}{(1-\delta_1)^2} - 1 = 0.384$
- $D_2 = \frac{1}{\delta_2^2} - 1 = 43.44$

1.6 Analog Lowpass Magnitude Response

Since we have to make a Butterworth filter, the general response is given as

$$H_{LPF}(s_L) H_{LPF}(-s_L) = \frac{1}{1 + \left(\frac{s_L}{j\Omega_c}\right)^{2N}}$$

The parameters N and Ω_c can be found as follows:

$$N = \left\lceil \frac{\log(\sqrt{D_2/D_1})}{\log(\Omega_{Ls}/\Omega_{Lp})} \right\rceil$$

and

$$\frac{\Omega_{Lp}}{D_1^{\frac{1}{2N}}} \leq \Omega_c \leq \frac{\Omega_{Ls}}{D_2^{\frac{1}{2N}}}$$

Using the above relations, we get

$$N = 8$$

$$1.0616 \leq \Omega_c \leq 1.096$$

We choose $\Omega_c = 1.0788$. Using these values, we have

$$H_{LPF}(s_L) H_{LPF}(-s_L) = \frac{1}{1 + \left(\frac{s_L}{j1.0788}\right)^{16}}$$

After using MATLAB to find and plot the poles, we have

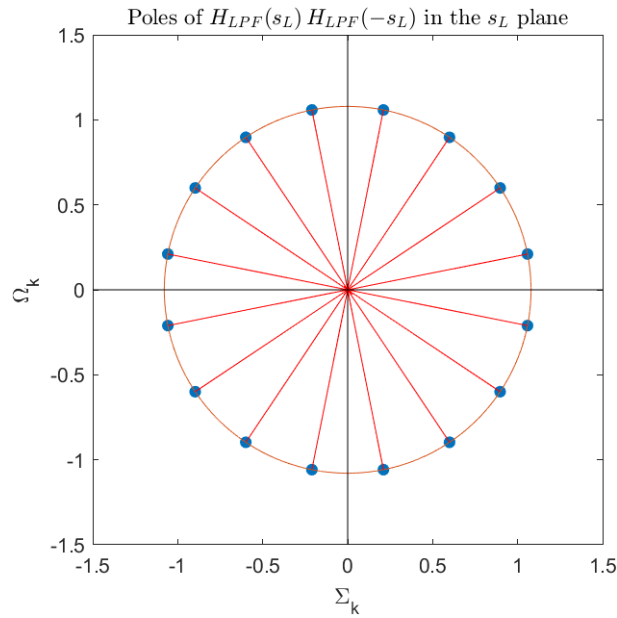


Figure 1.1: Poles of the equivalent lowpass filter

To have a realisable filter, we choose only those poles which lie on the left half of the complex s_L plane, which are as follows:

Pole	Value
s_1	$-0.5993 + j 0.8970$
s_2	$-0.8970 + j 0.5993$
s_3	$-1.0581 + j 0.2105$
s_4	$-1.0581 - j 0.2105$
s_5	$-0.8970 - j 0.5993$
s_6	$-0.5993 - j 0.8970$
s_7	$-0.2105 - j 1.0581$
s_8	$-0.2105 + j 1.0581$

Table 1.1: Poles on left half plane

Using the poles determined in table 1.1, the **equivalent analog lowpass transfer function** is given by

$$H_{LPF}(s_L) = \frac{\Omega_c^N}{\prod_{k=1}^N (s_L - s_k)} = \frac{1.8345}{\sum_{k=0}^8 a_k s_L^k}$$

where the coefficients of the denominator are

Coefficient	Value
a_0	1.8345
a_1	8.7167
a_2	20.708
a_3	31.921
a_4	34.794
a_5	27.428
a_6	15.289
a_7	5.5297
a_8	1

Table 1.2: Coefficients of Denominator

The magnitude and the phase responses of this equivalent Butterworth LPF are as follows:

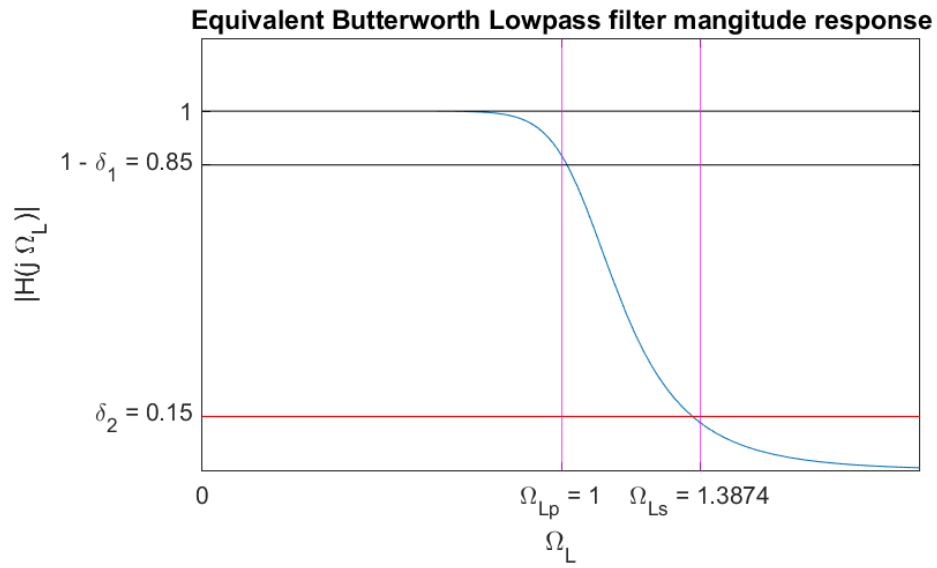


Figure 1.2: Magnitude Response

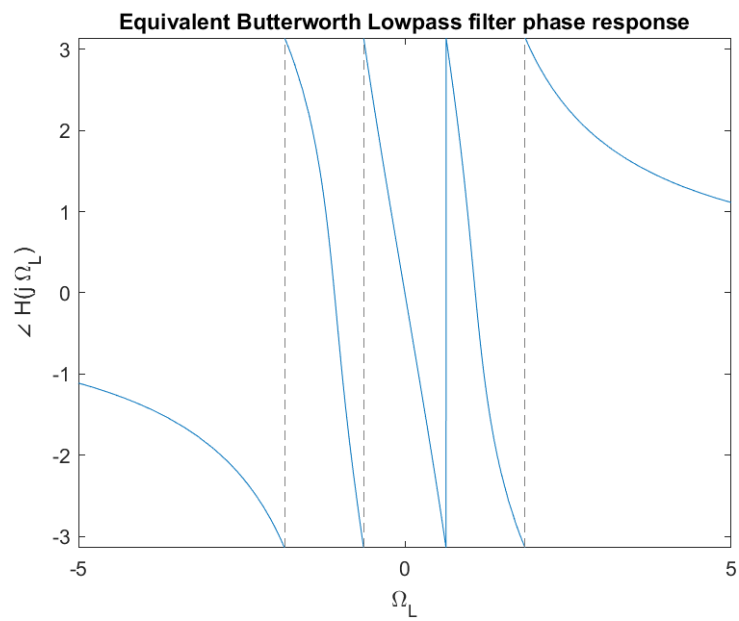


Figure 1.3: Phase Response

1.7 Analog Bandpass Transfer Function

We now use the inverse transformation to convert the equivalent Butterworth lowpass transfer function to get the Bandpass transfer function using the relation:

$$s_L \leftarrow \frac{s^2 + \Omega_0^2}{B s}$$

where B and Ω_0 have the values as we found in section 1.4

$$H_{LPPF}\left(\frac{s^2 + \Omega_0^2}{B s}\right) = H_{BPF}(s) = \frac{1.8345}{168879.9214 + \sum_{k=-8}^8 a_k s^k}$$

The coefficients a_k are as follows:

Coefficient	Value
a_1	182483.5084
a_{-1}	94310.6177
a_2	252765.02515
a_{-2}	67513.5454
a_3	204166.44531
a_{-3}	28183.5223
a_4	220581.2811
a_{-4}	15736.8035
a_5	122303.3771
a_{-5}	4509.4469
a_6	101839.6751
a_{-6}	1940.6124
a_7	30259.6314
a_{-7}	298.0042
a_8	18714.6217
a_{-8}	95.2525

Table 1.3: Coefficients of Denominator in the Analog BP transfer function

The magnitude and frequency response are as follows:

Please see the next page

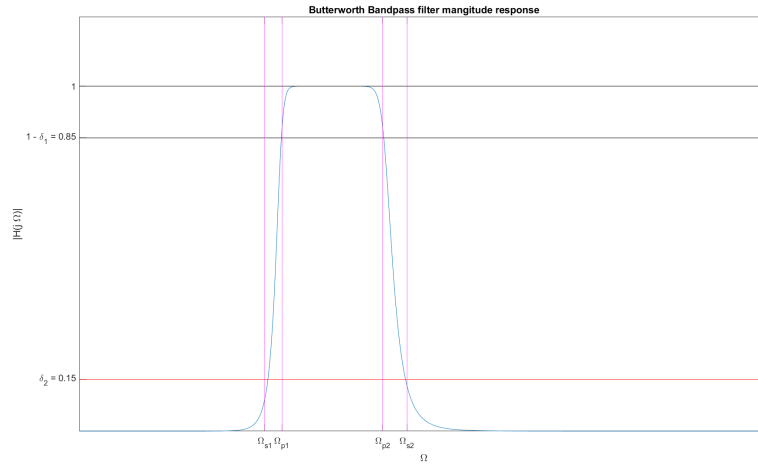


Figure 1.4: Magnitude response of the analog BPF

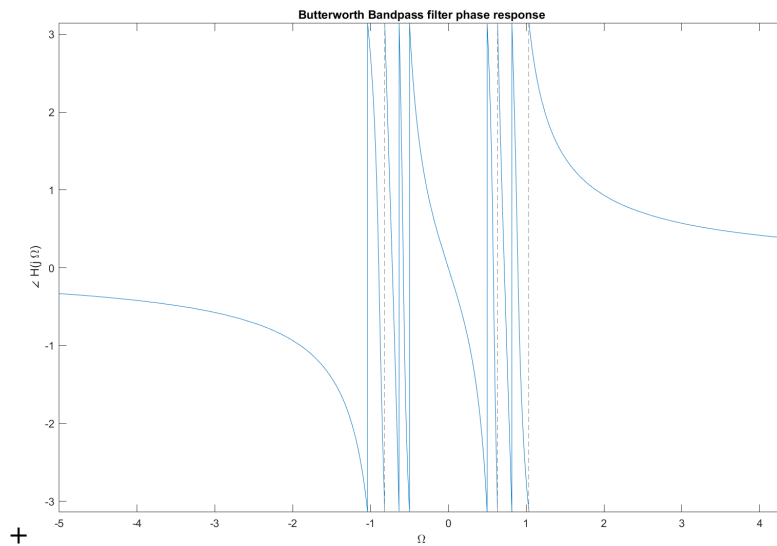


Figure 1.5: Phase response of the analog BPF

1.8 Discrete-Time Bandpass Transfer Function

We now make use of the Bilinear Transformation to convert the analog bandpass filter into a discrete bandpass filter in the normalized angular frequency domain.

The transform is as follows:

$$s \leftarrow \frac{1 - z^{-1}}{1 + z^{-1}}$$

The discrete time filter transfer function is

$$H_{BPF}\left(\frac{1 - z^{-1}}{1 + z^{-1}}\right) = H(z) = \frac{\sum_{k=0}^{16} b_k z^k}{\sum_{k=0}^{16} a_k z^k}$$

with coefficients

Coefficient	Value
a_{16}	0.0083
a_{15}	-0.0366
a_{14}	0.1197
a_{13}	-0.2688
a_{12}	0.5003
a_{11}	-0.7489
a_{10}	0.9662
a_9	-1.0495
a_8	0.9966
a_7	-0.8059
a_6	0.5697
a_5	-0.3389
a_4	0.1737
a_3	-0.0715
a_2	0.0244
a_1	-0.0057
a_0	0.0010

Table 1.4: Denominator ($\times 10^8$)

Coefficient	Value
b_{16}	1
b_{15}	0
b_{14}	-8
b_{13}	0
b_{12}	28
b_{11}	0
b_{10}	-56
b_9	0
b_8	70
b_7	0
b_6	-56
b_5	0
b_4	28
b_3	0
b_2	-8
b_1	0
b_0	1

Table 1.5: Numerator

Please see the next page

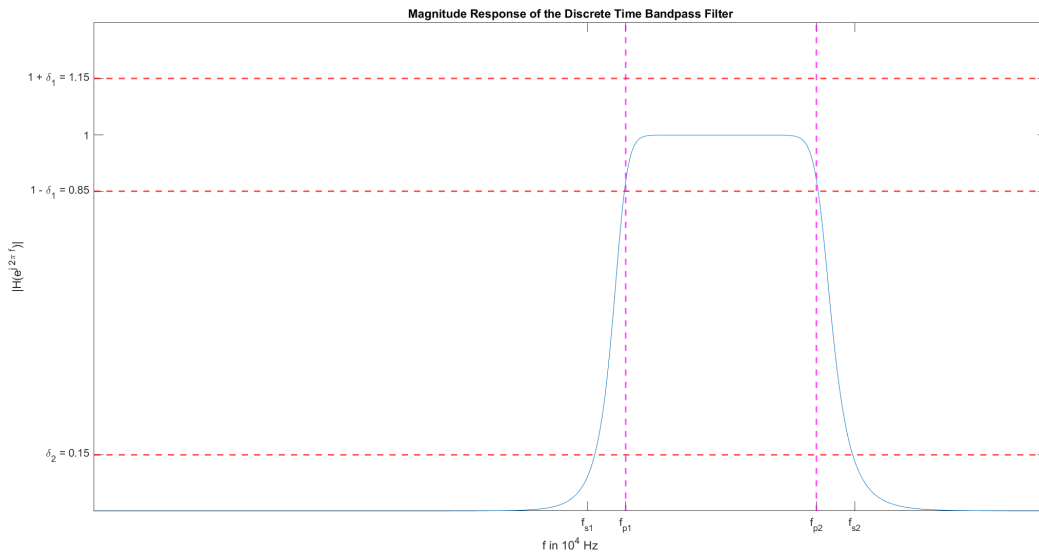


Figure 1.6: Magnitude Response of Discrete Time Bandpass Filter

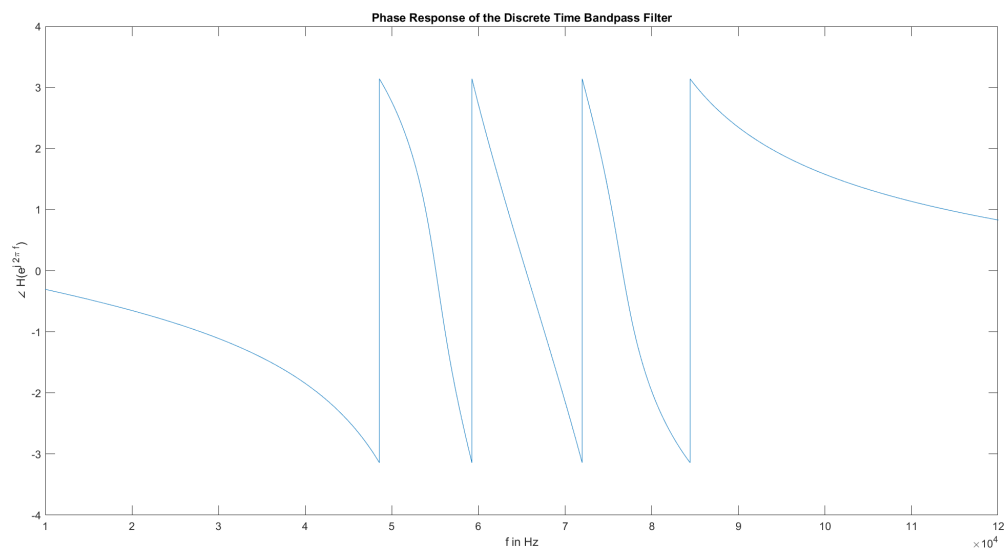


Figure 1.7: Phase Response of Discrete Time Bandpass Filter

2 Finite Impulse Response Bandpass Filter

In this part, we will use the windowing method to design a finite impulse response bandpass filter for the filter 44 using the same specifications as above. Recalling the specifications from section 1.2, we have the following normalized specifications:

- $\omega_{p1} = 2\pi \frac{55.8}{330} = 0.3382 \pi$
- $\omega_{p2} = 2\pi \frac{75.8}{330} = 0.4594 \pi$
- Transition bandwidth($\Delta\omega_T$) = $2\pi \frac{4}{330} = 0.0242 \pi$
- $\omega_{s1} = \omega_{p1} - 0.0242 \pi = 0.314 \pi$
- $\omega_{s2} = \omega_{p2} + 0.0242 \pi = 0.4836 \pi$
- Passband and Stopband tolerance(δ) = 0.15

We will use a time-shifted **Kaiser Window** and multiply it with the impulse response of an ideal bandpass filter to get a realizable FIR filter.

2.1 Implementation Technique

While calculating the ideal bandpass filter response, the mean of the lower passband and stopband edges are used as one of the lower passband edge and the mean of the upper passband and stopband edges are used to find the upper passband edge. The same is mentioned in the following figure:

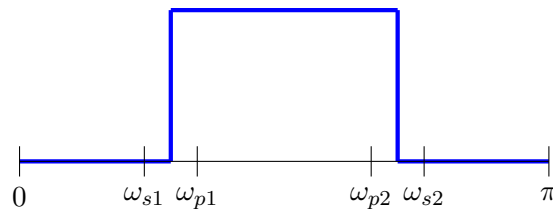


Figure 2.1: Ideal Bandpass Filter Magnitude Response

2.2 Kaiser Window Parameters

We have

$$A = -20 \log_{10}(\delta) = 16.478$$

As $A < 21$, the shape parameter $\beta = 0$ and the Kaiser window in this case is essentially a rectangular window. Also, the window width M is

$$M \geq \left\lceil 1 + \frac{A - 8}{2.285 \Delta\omega_T} \right\rceil = 50$$

After tuning by hit and trail method so that the maximum error is within the tolerance, it has been found that a window width of $M = 66$ satisfies all the required specifications.

2.3 Discrete Time FIR Filter Transfer Function

The transfer function of the FIR filter obtained after adjusting for causality is as shown below. These are the coefficients of the Z transform of the impulse response of the FIR filter.

```

Columns 1 through 8
-0.0177 -0.0031 0.0115 0.0070 -0.0020 0.0001 0.0012 -0.0095
Columns 9 through 16
-0.0124 0.0084 0.0257 0.0067 -0.0264 -0.0241 0.0114 0.0274
Columns 17 through 24
0.0056 -0.0134 -0.0058 -0.0010 -0.0144 -0.0073 0.0341 0.0436
Columns 25 through 32
-0.0237 -0.0840 -0.0279 0.0904 0.0968 -0.0419 -0.1376 -0.0433
Columns 33 through 40
0.1175 0.1175 -0.0433 -0.1376 -0.0419 0.0968 0.0904 -0.0279
Columns 41 through 48
-0.0840 -0.0237 0.0436 0.0341 -0.0073 -0.0144 -0.0010 -0.0058
Columns 49 through 56
-0.0134 0.0056 0.0274 0.0114 -0.0241 -0.0264 0.0067 0.0257
Columns 57 through 64
0.0084 -0.0124 -0.0095 0.0012 0.0001 -0.0020 0.0070 0.0115
Columns 65 through 66
-0.0031 -0.0177

```

Figure 2.2: Bandpass FIR Transfer Function

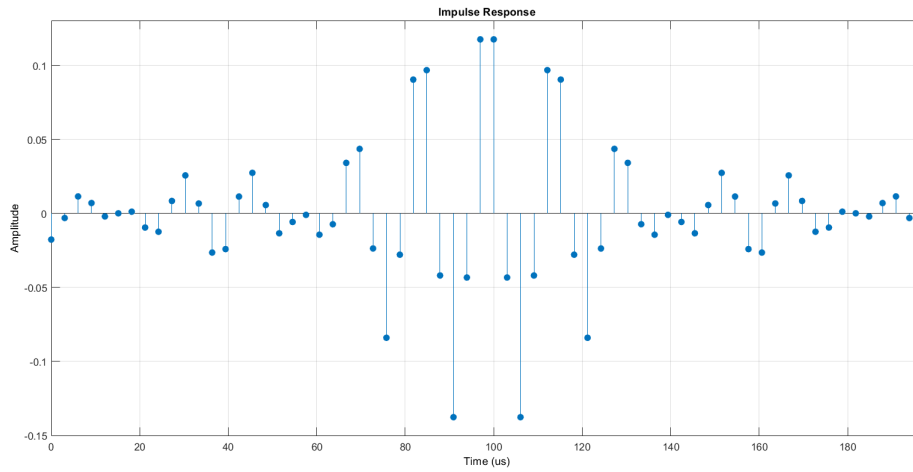


Figure 2.3: Bandpass FIR Impulse Response

2.4 Magnitude and Phase Response

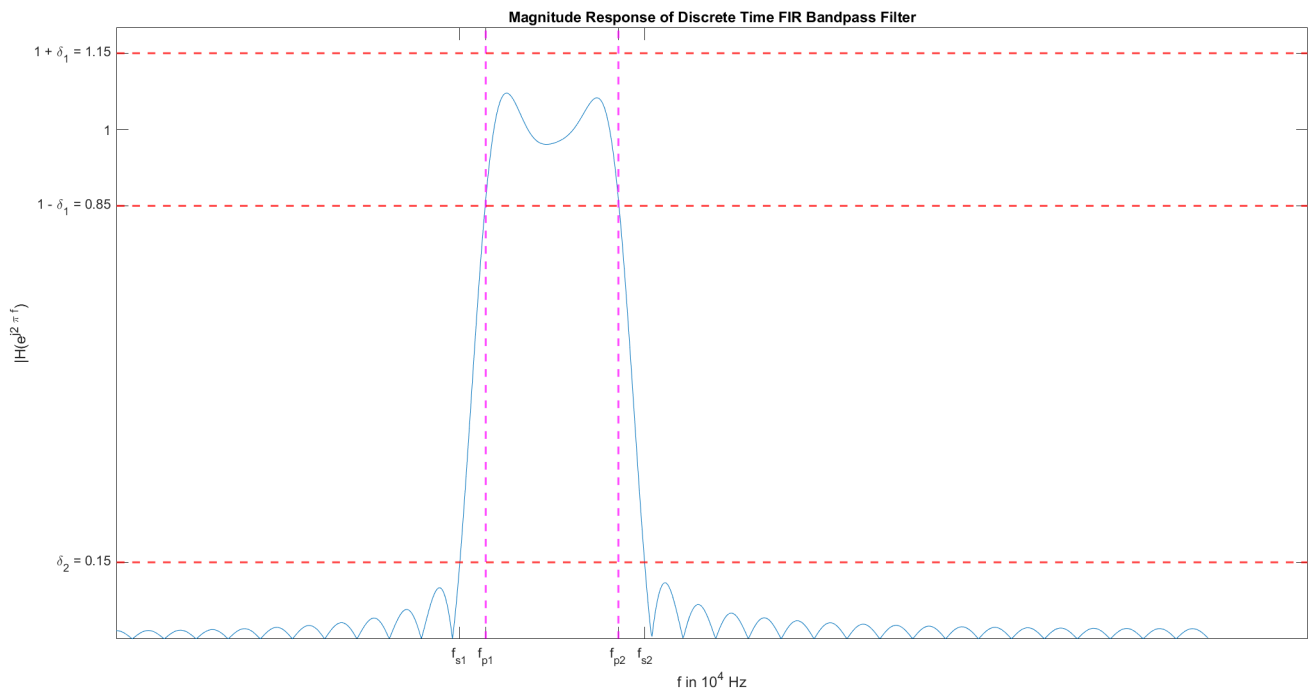


Figure 2.4: Magnitude Response of the FIR filter

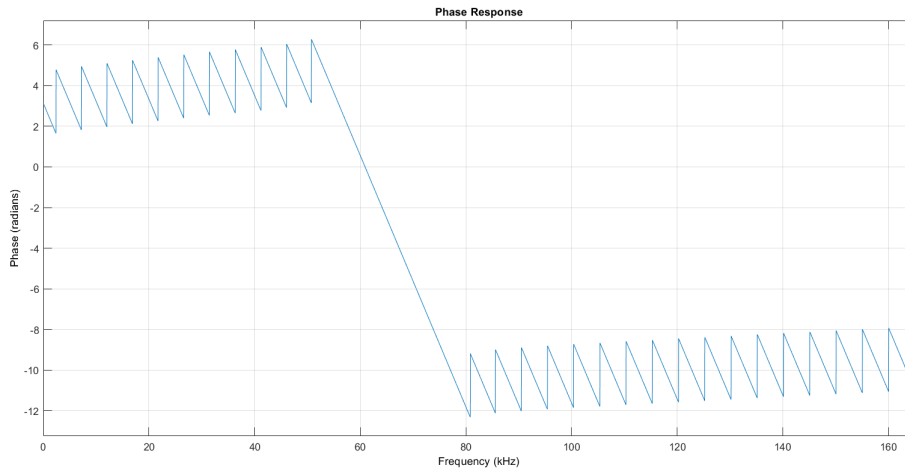


Figure 2.5: Phase Response of the FIR filter

2.5 Comparison with the IIR Filter

We can see that on comparing figure 1.7 and figure 2.5, we can see that the FIR filter gives a linear phase response for the frequencies in the passband range i.e from 55.8kHz to 75.8kHz.

Also, we can observe that the order of the IIR filter is 8 and the order of the FIR filter is 66 which is much higher. This once again proves the fact that *there is no free lunch*, i.e., we are getting a linear phase response at the expense of resources.

3 Infinite Impulse Response Bandstop Filter

The filter number $m = 44$. Hence,

$$q(m) = \lfloor 0.1m \rfloor = 4$$

$$r(m) = m - 10q(m) = 44 - 40 = 4$$

$$BL(m) = 25 + 1.9q(m) + 4.1r(m) = 49 \text{ kHz}$$

$$BH(m) = BL(m) + 20 = 69 \text{ kHz}$$

3.1 Unnormalized Specifications

- Lower Passband Edge = 45 kHz
- Upper passband Edge = 73 kHz
- Transition band width = 4 kHz
- Lower Stopband Edge = 49 kHz
- Upper Stopband Edge = 69 kHz
- Passband tolerance = 0.15
- Stopband tolerance = 0.15
- Passband nature = equiripple
- Stopband nature = monotonic
- Sampling frequency = 260 kHz

3.2 Normalized Specifications

The sampling angular frequency $2\pi f_s$ should map to 2π after normalization. Hence the transformation used is

$$\omega = 2\pi \frac{f_s}{f}$$

The normalized specifications are as follows (following the notations used in the lecture)

- $\omega_{p1} = 2\pi \frac{45}{260} = 0.3461 \pi$
- $\omega_{p2} = 2\pi \frac{73}{260} = 0.5615 \pi$
- $\omega_{s1} = 2\pi \frac{49}{260} = 0.3769 \pi$
- $\omega_{s2} = 2\pi \frac{69}{260} = 0.5307 \pi$
- Transition bandwidth = $2\pi \frac{4}{260} = 0.0308 \pi$
- Passband and Stopband tolerance = 0.15
- Passband nature = equiripple
- Stopband nature = monotonic

3.3 Analog Filter Specifications using Bilinear Transform

We use the following transformation to move from the z domain to the s domain.

$$s = \frac{1 - z^{-1}}{1 + z^{-1}}$$

If we put $z = j\omega$ and $s = j\Omega$, we get

$$\Omega = \tan \frac{\omega}{2}$$

The specifications now are

- Passband nature = equiripple
- Stopband nature = monotonic
- $\Omega_{s1} = 0.6725$
- $\Omega_{s2} = 1.1014$
- $\Omega_{p1} = 0.6044$
- $\Omega_{p2} = 1.2146$
- $\delta_1 = \delta_2 = 0.15$

3.4 Analog Frequency Transformation

Here we use the **Bandstop to Lowpass** analog frequency transformation, which is as follows:

$$\Omega_L = \frac{B \Omega}{\Omega_0^2 - \Omega^2}$$

where

$$\Omega_0 = \sqrt{\Omega_{p1} \Omega_{p2}} = 0.8568$$

and

$$B = \Omega_{p2} - \Omega_{p1} = 0.6102$$

Using the transformation mentioned above, we now have

- Passband and Stopband nature = monotonic
- $\Omega_{Ls1} = 1.456$
- $\Omega_{Ls2} = -1.4031$
- $\Omega_{Lp1} = +1$
- $\Omega_{Lp2} = -1$
- $\delta_1 = \delta_2 = 0.15$

3.5 Equivalent Analog Lowpass Filter Specifications

Using the more stringent specifications from those obtained in section 3.4, we have

- Passband nature = equiripple
- Stopband nature = passband
- $\Omega_{Ls} = \min\{|\Omega_{Ls2}|, |\Omega_{Ls1}|\} = 1.4031$
- $\Omega_{Lp} = +1$
- $\delta_1 = \delta_2 = 0.15$
- $D_1 = \frac{1}{(1-\delta_1)^2} - 1 = 0.384$
- $D_2 = \frac{1}{\delta_2^2} - 1 = 43.44$

3.6 Analog Lowpass Magnitude Response

Since we have to make a Chebyshev filter, the general response is given as

$$H_{LPF}(s_L) H_{LPF}(-s_L) = \frac{1}{1 + \epsilon^2 C_N^2(s_L/j\Omega_{Lp})}$$

Where the parameters ϵ and N are given by

$$\epsilon = \sqrt{D_1} = 0.6197$$

and

$$N = \left\lceil \frac{\cosh^{-1}(\sqrt{D_2/D_1})}{\cosh^{-1}(\Omega_{Ls}/\Omega_{Lp})} \right\rceil = 4$$

After using MATLAB to find and plot the poles, we have

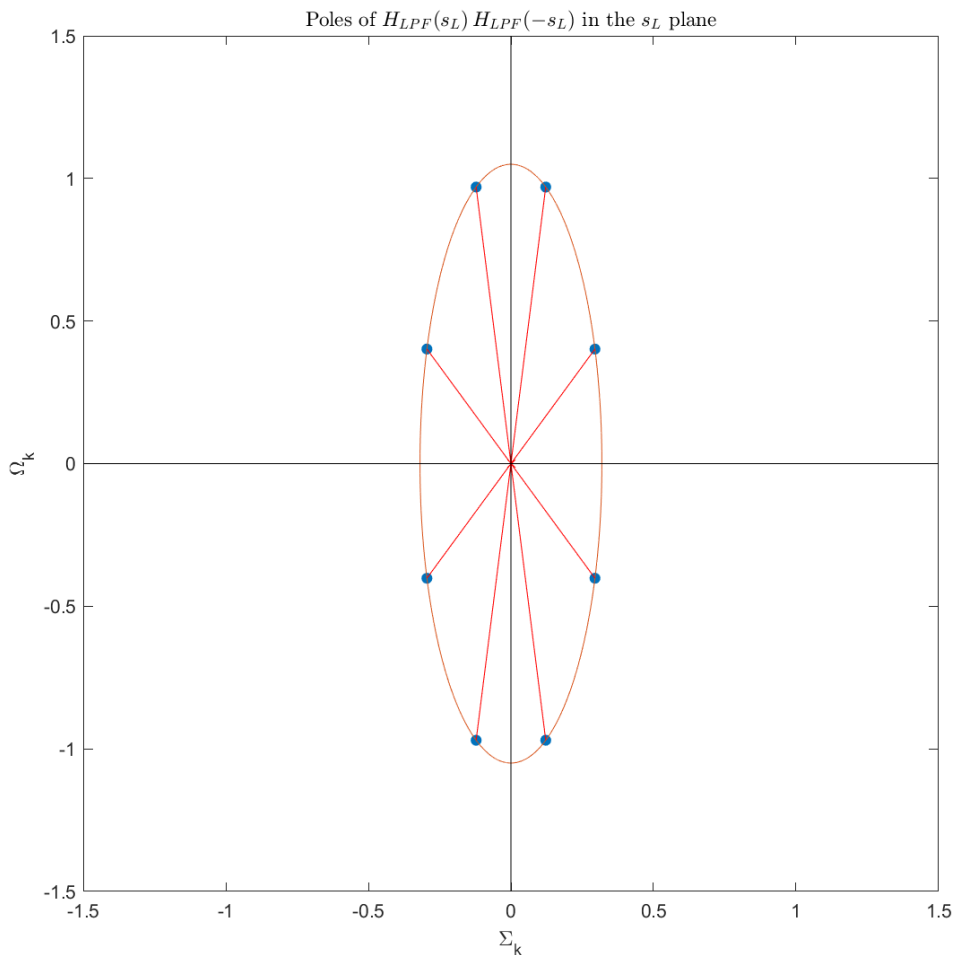


Figure 3.1: Poles of the equivalent lowpass filter

To have a realisable filter, we choose only those poles which lie on the left half of

the complex s_L plane, which are as follows:

Pole	Value
s_1	$-0.1222 + j 0.9698$
s_2	$-0.2949 + j 0.4017$
s_3	$-0.2949 - j 0.4017$
s_4	$-0.1222 - j 0.9698$

Table 3.1: Poles on left half plane

Using the poles determined in table 3.1, the **equivalent analog lowpass transfer function** is given by

$$H_{LPF}(s_L) = \frac{A}{\prod_{k=1}^N (s_L - s_k)}$$

Substituting the constraint that the value of the magnitude response at the passband edge is $1 - \delta_1$, we have

$$H_{LPF}(s_L) = \frac{\prod_{i=1}^4 s_i}{\sqrt{1 + D_1} \left(\prod_{k=1}^N (s_L - s_k) \right)} = \frac{0.2373}{1.1764 s_L^4 + 0.9814 s_L^3 + 1.5858 s_L^2 + 0.7345 s_L + 0.2792}$$

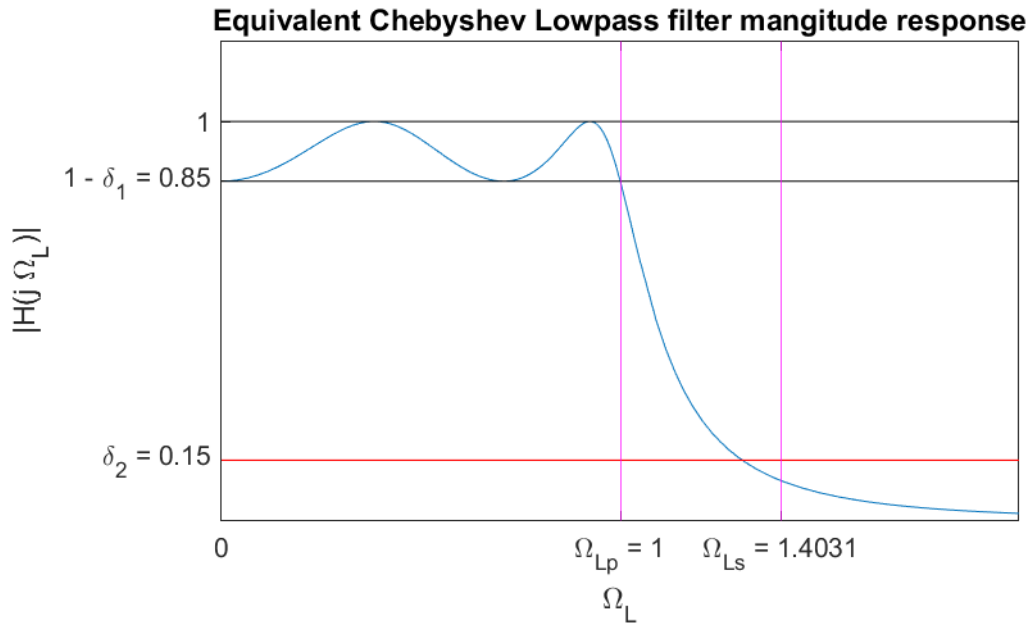


Figure 3.2: Magnitude Response

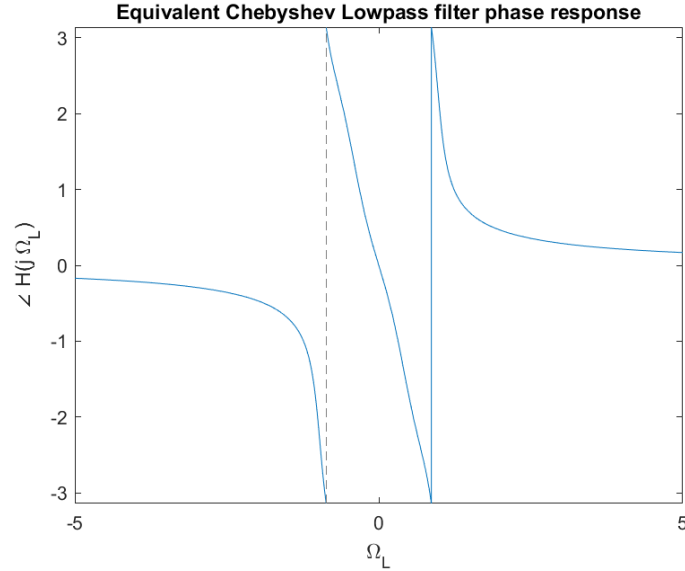


Figure 3.3: Phase Response

3.7 Analog Bandstop Transfer Function

We now use the inverse transformation to convert the equivalent Chebyshev lowpass transfer function to get the Bandstop transfer function using the relation:

$$s_L \leftarrow \frac{B s}{s^2 + \Omega_0^2}$$

where B and Ω_0 have the values as we found in section 3.4

$$H_{LPF}\left(\frac{B s}{s^2 + \Omega_0^2}\right) = H_{BSF}(s) = \frac{1.3516 s^8 + 3.9691 s^6 + 4.3707 s^4 + 2.1390 s^2 + 3.9257}{4.6183 + \sum_{k=1}^8 b_k s^k}$$

Coefficient	Value
b_8	1.5902
b_7	2.5527
b_6	8.0327
b_5	6.8921
b_4	1.1001
b_3	5.0595
b_2	4.3289
b_1	1.0099

Table 3.2: Denominator

The magnitude and frequency response are as follows:

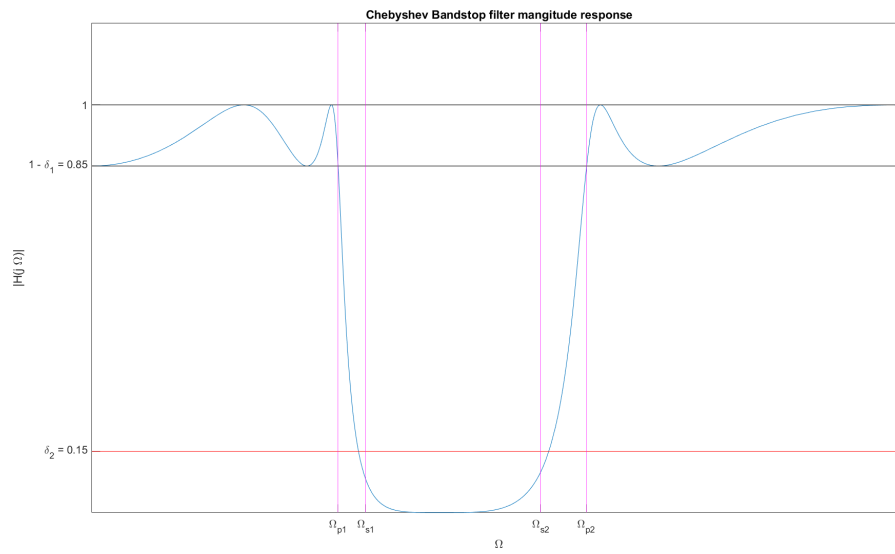


Figure 3.4: Magnitude Response of Chebyshev Bandstop filter

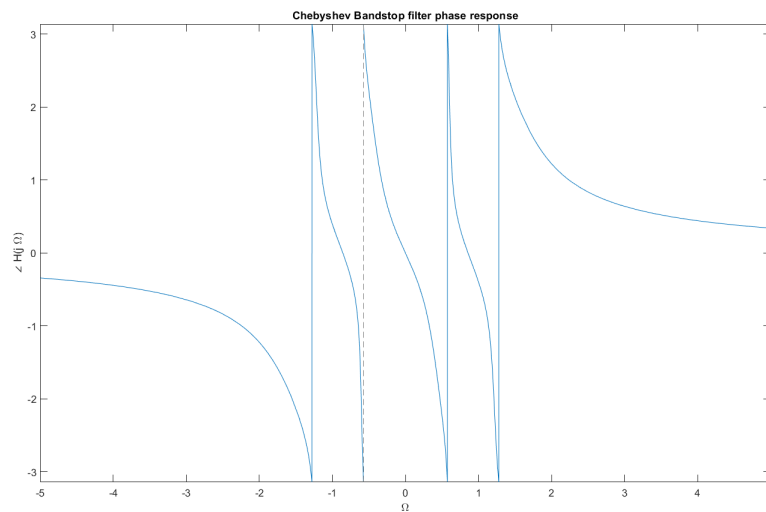


Figure 3.5: Phase Response of Chebyshev Bandstop filter

3.8 Discrete-Time Bandstop Transfer Function

We now make use of the Bilinear Transformation to convert the analog bandstop filter into a discrete bandstop filter in the normalized angular frequency domain.

The transform is as follows:

$$s \longleftarrow \frac{1 - z^{-1}}{1 + z^{-1}}$$

The discrete time filter transfer function is

$$H_{BSF}\left(\frac{1 - z^{-1}}{1 + z^{-1}}\right) = H(z) = \frac{\sum_{k=0}^8 b_k z^k}{\sum_{k=0}^8 a_k z^k}$$

Coefficient	Value
a_8	4.0936
a_7	-3.6764
a_6	8.8842
a_5	-5.8977
a_4	8.6077
a_3	-3.7768
a_2	3.6893
a_1	-1.0919
a_0	9.9081

Table 3.3: Denominator

Coefficient	Value
b_8	1.2223
b_7	-1.4993
b_6	5.5789
b_5	-4.6390
b_4	8.7240
b_3	-4.6390
b_2	5.5789
b_1	-1.4993
b_0	1.2223

Table 3.4: Numerator

Please see the next page

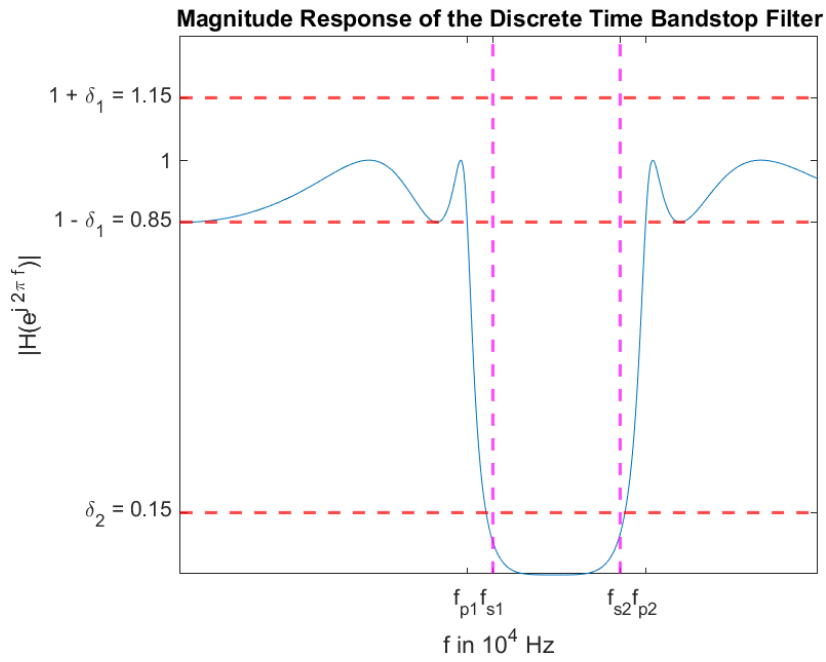


Figure 3.6: Magnitude Response of Discrete Time Bandstop Filter

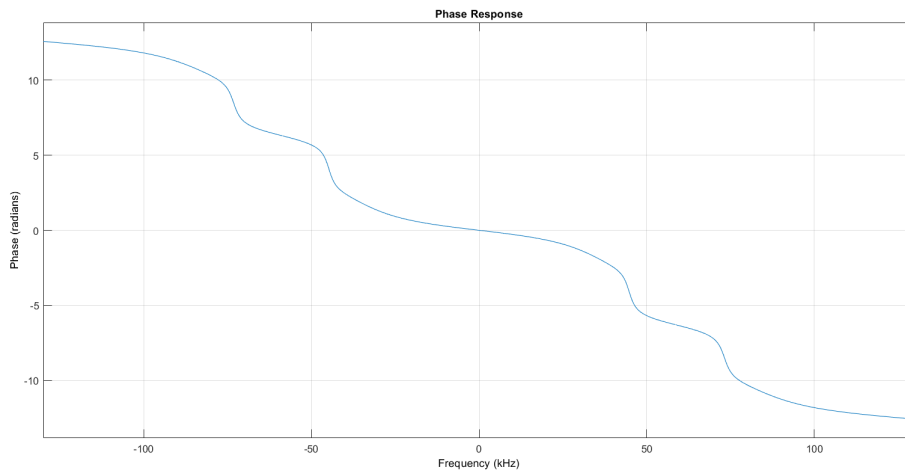


Figure 3.7: Phase Response of Discrete Time Bandstop Filter

4 Finite Impulse Response Bandstop Filter

In this part, we will use the windowing method to design a finite impulse response bandstop filter for the filter 44 using the same specifications as above. Recalling the specifications from section 3.2, we have the following normalized specifications:

- $\omega_{p1} = 2\pi \frac{45}{260} = 0.3461 \pi$
- $\omega_{p2} = 2\pi \frac{73}{260} = 0.5615 \pi$
- $\omega_{s1} = 2\pi \frac{49}{260} = 0.3769 \pi$
- $\omega_{s2} = 2\pi \frac{69}{260} = 0.5307 \pi$
- Transition bandwidth($\Delta\omega_T$) = $2\pi \frac{4}{260} = 0.0308 \pi$
- Passband and Stopband tolerance = 0.15

We will use a time-shifted **Kaiser Window** and multiply it with the impulse response of an ideal bandstop filter to get a realizable FIR filter.

4.1 Implementation Technique

While calculating the ideal bandstop filter response, the mean of the lower passband and stopband edges are used as one of the lower stopband edge and the mean of the upper passband and stopband edges are used to find the upper stopband edge. The same is mentioned in the following figure:

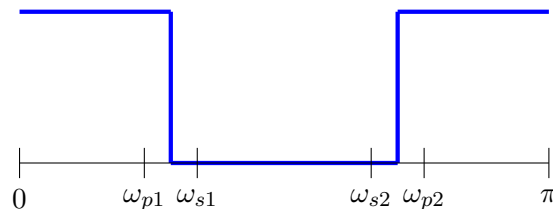


Figure 4.1: Ideal Bandstop Filter Magnitude Response

4.2 Kaiser Window Parameters

We have

$$A = -20 \log_{10}(\delta) = 16.478$$

As $A < 21$, the shape parameter $\beta = 0$ and the Kaiser window in this case is essentially a rectangular window. Also, the window width M is

$$M \geq \left\lceil 1 + \frac{A - 8}{2.285 \Delta\omega_T} \right\rceil = 40$$

After tuning by hit and trail method so that the maximum error is within the tolerance, it has been found that a window width of $M = 53$ satisfies all the required specifications.

4.3 Discrete Time FIR Filter Transfer Function

The transfer function of the FIR filter obtained after adjusting for causality is as shown below. These are the coefficients of the Z transform of the impulse response of the FIR filter.

```
Columns 1 through 8
-0.0188    0.0098    0.0156   -0.0020   -0.0028    0.0005   -0.0144   -0.0088
Columns 9 through 16
 0.0267    0.0228   -0.0271   -0.0326    0.0161    0.0273   -0.0030   -0.0028
Columns 17 through 24
 0.0018   -0.0346   -0.0232    0.0693    0.0674   -0.0839   -0.1220    0.0684
Columns 25 through 32
 0.1671   -0.0263    0.8154   -0.0263    0.1671    0.0684   -0.1220   -0.0839
Columns 33 through 40
 0.0674    0.0693   -0.0232   -0.0346    0.0018   -0.0028   -0.0030    0.0273
Columns 41 through 48
 0.0161   -0.0326   -0.0271    0.0228    0.0267   -0.0088   -0.0144    0.0005
Columns 49 through 53
-0.0028   -0.0020    0.0156    0.0098   -0.0188
```

Figure 4.2: Bandstop FIR Transfer Function

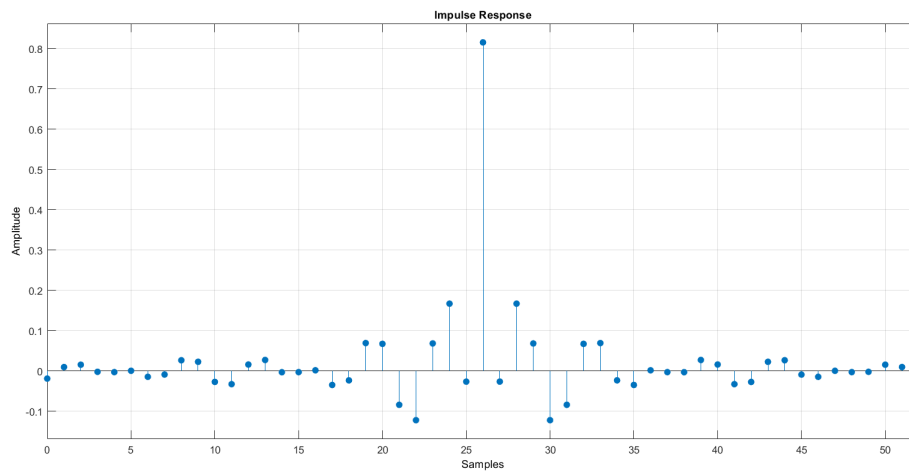


Figure 4.3: Bandstop FIR Impulse Response

4.4 Magnitude and Phase Response

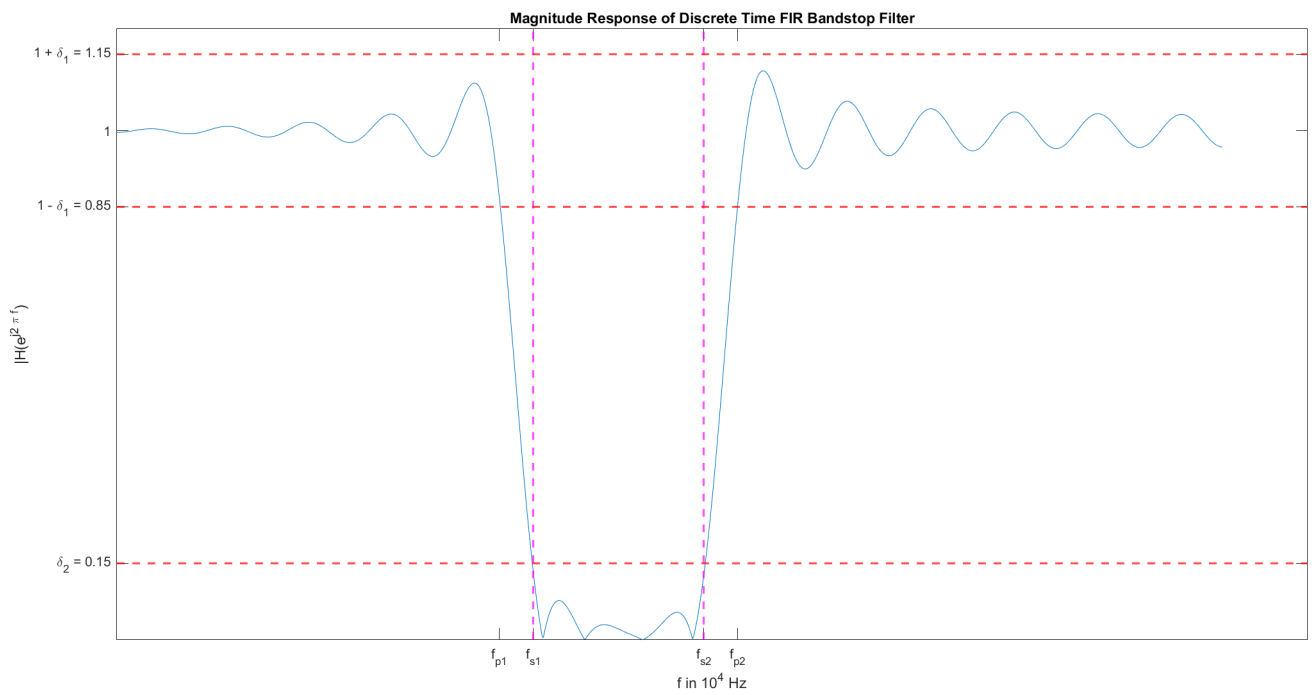


Figure 4.4: Magnitude Response of the FIR filter

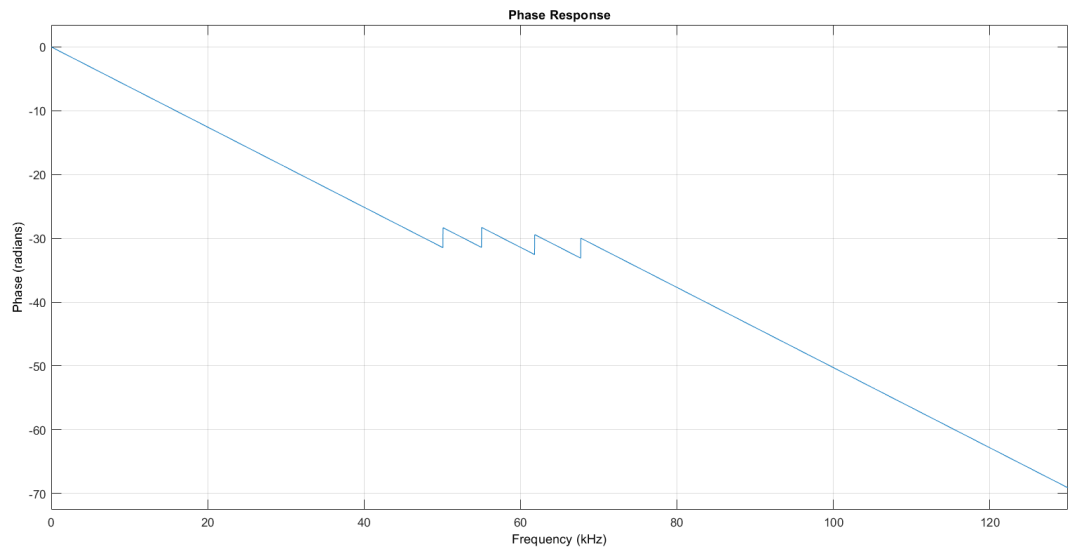


Figure 4.5: Phase Response of the FIR filter

4.5 Comparison with the IIR Filter

We can see that on comparing figure 3.7 and figure 4.5, we can see that the FIR filter gives a linear phase response for the frequencies in the passband range.

Also, we can observe that the order of the IIR filter is 4 and the order of the FIR filter is 53 which is much higher. This once again proves the fact that *there is no free lunch*, i.e., we are getting a linear phase response at the expense of resources.

5 Elliptical Bandpass filter design

The filter number $m = 44$. Hence,

$$q(m) = \lfloor 0.1m \rfloor = 4$$

$$r(m) = m - 10q(m) = 44 - 40 = 4$$

$$BL(m) = 25 + 1.7q(m) + 6.1r(m) = 55.8 \text{ kHz}$$

$$BH(m) = BL(m) + 20 = 75.8 \text{ kHz}$$

5.1 Unnormalized Specifications

- Lower Passband Edge = 55.8 kHz
- Upper passband Edge = 75.8 kHz
- Transition band width = 4 kHz
- Passband tolerance = 0.15
- Stopband tolerance = 0.15
- Passband nature = monotonic
- Stopband nature = monotonic
- Sampling frequency = 330 kHz

5.2 Normalized Specifications

The sampling angular frequency $2\pi f_s$ should map to 2π after normalization. Hence the transformation used is

$$\omega = 2\pi \frac{f_s}{f}$$

The normalized specifications are as follows (following the notations used in the lecture)

- $\omega_{p1} = 2\pi \frac{55.8}{330} = 0.3382 \pi$
- $\omega_{p2} = 2\pi \frac{75.8}{330} = 0.4594 \pi$

- Transition bandwidth = $2\pi \frac{4}{330} = 0.0242 \pi$
- $\omega_{s1} = \omega_{p1} - 0.0242 \pi = 0.314 \pi$
- $\omega_{s2} = \omega_{p2} + 0.0242 \pi = 0.4836 \pi$
- Passband and Stopband tolerance = 0.15
- Passband and Stopband nature = equiripple

5.3 Analog Filter Specifications using Bilinear Transform

We use the following transformation to move from the z domain to the s domain.

$$s = \frac{1 - z^{-1}}{1 + z^{-1}}$$

If we put $z = j\omega$ and $s = j\Omega$, we get

$$\Omega = \tan \frac{\omega}{2}$$

The specifications now are

- Passband and Stopband nature = monotonic
- $\Omega_{s1} = 0.5375$
- $\Omega_{s2} = 0.9498$
- $\Omega_{p1} = 0.5875$
- $\Omega_{p2} = 0.8799$
- $\delta_1 = \delta_2 = 0.15$

.

5.4 Analog Frequency Transformation

Here we use the **Bandpass to Lowpass** analog frequency transformation, which is as follows:

$$\Omega_L = \frac{\Omega^2 - \Omega_0^2}{B\Omega}$$

where

$$\Omega_0 = \sqrt{\Omega_{p1} \Omega_{p2}} = 0.7189$$

and

$$B = \Omega_{p2} - \Omega_{p1} = 0.2924$$

Using the transformation mentioned above, we now have

- Passband and Stopband nature = monotonic
- $\Omega_{Ls1} = -1.4501$
- $\Omega_{Ls2} = 1.3874$
- $\Omega_{Lp1} = -1$
- $\Omega_{Lp2} = +1$
- $\delta_1 = \delta_2 = 0.15$

5.5 Equivalent Analog Lowpass Filter Specifications

Using the more stringent specifications from those obtained in section 1.4, we have

- Passband and Stopband nature = monotonic
- $\Omega_{Ls} = \min\{\Omega_{Ls2}, |\Omega_{Ls1}|\} = 1.3874$
- $\Omega_{Lp} = +1$
- $\delta_1 = \delta_2 = 0.15$
- $D_1 = \frac{1}{(1-\delta_1)^2} - 1 = 0.384$
- $D_2 = \frac{1}{\delta_2^2} - 1 = 43.44$

5.6 Analog Lowpass Magnitude Response

The general transfer function for an **Elliptic** filter is given by

$$H_{LPF}(s_L) H_{LPF}(-s_L) = \frac{1}{1 + \epsilon^2 F_N^2\left(\frac{s_L}{j\Omega_{Lp}}\right)}$$

with

$$F_N(w) = \text{cd}(NuK_1, k_1)$$

and

$$w = \text{cd}(uK, k)$$

where $\text{cd}(x, k)$ denotes the Jacobian elliptic function cd with modulus k and real quarter-period K .

5.6.1 Jacobian Elliptic Functions

The elliptic function $w = \text{sn}(z, k)$ is defined as follows:

$$z = \int_0^\phi \frac{d\theta}{\sqrt{1 - k^2 \sin^2 \theta}} = \int_0^w \frac{dt}{\sqrt{(1 - t^2)(1 - k^2 t^2)}} \quad , w = \text{sn}(\phi(z, k))$$

There are three more relevant elliptic functions, cn , dn , and cd defined as:

$$\begin{aligned} w &= \text{cn}(z, k) = \cos(\phi(z, k)) \\ w &= \text{dn}(z, k) = \sqrt{1 - k^2 \text{sn}^2(z, k)} \\ w &= \text{cd}(z, k) = \frac{\text{cn}(z, k)}{\text{dn}(z, k)} \end{aligned}$$

We define the *complete elliptic integral of first kind* $K(k)$ or simply K as the value of z when $\text{sn}(z, k) = 1$ i.e. when $\phi = \pi/2$

$$K(k) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - k^2 \sin^2 \theta}}$$

Clearly $\text{cd}(K, k) = 0$. Associated with an elliptic modulus k , there is a complementary modulus $k' = \sqrt{1 - k^2}$ and its associated complete elliptic integral $K(k')$ denoted by $K'(k)$ or simply K'

$$K'(k) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - k'^2 \sin^2 \theta}} = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - (1 - k^2) \sin^2 \theta}}$$

The significance of K and K' is that sn and cd are *doubly-periodic functions* in the z plane with a real period $4K$ and a complex period $2jK'$. Some of the useful properties are as follows:

$$\text{cd}(z + (2n - 1)K, k) = (-1)^n \text{sn}(z, k) \quad (1)$$

$$\text{cd}(z + 2nK, k) = (-1)^n \text{cd}(z, k) \quad (2)$$

$$\text{cd}(z + jK', k) = \frac{1}{k \text{cd}(z, k)} \quad (3)$$

$$\text{cd}(jz, k) = \frac{1}{\text{dn}(z, k')} \quad (4)$$

$$\text{cd}(jK', k) = \frac{1}{k} \quad (5)$$

$\text{cd}(z, k)$ has zeroes at $(2m + 1)K + j2nK'$ and poles at $(2m + 1)K + j(2n + 1)K'$ for any integers m and n in the z plane.

5.6.2 Elliptic Filter Parameters

The order of the filter N is determining by putting a constraint that the magnitude response at the passband edge is not less than $1 - \delta_1$ and at the stopband edge not greater than δ_2 .

$$N = \left\lceil \frac{K K'_1}{K' K_1} \right\rceil$$

where K, K', K_1, K'_1 are defined for the complete elliptic integrals of modulus k and k_1 given by

$$k = \frac{\Omega_{Lp}}{\Omega_{Ls}} = 0.72077$$

$$k_1 = \sqrt{\frac{D_1}{D_2}} = 0.0940$$

We then have

Parameter	Value
K	1.8710
K'	1.8379
K_1	1.5743
K'_1	3.7566
N	3

Table 5.1: Elliptic Filter Parameters

We now have to find the poles of the the transfer function. The poles of the transfer function are given by the zeros of the denominator and the zeros of the transfer function are given by the poles of $F_N(\frac{s_L}{j\Omega_{Lp}})$. The zeros and poles of the transfer function are given by

$$\begin{aligned} z_i &= j \Omega_{Lp} (k\zeta_i)^{-1}, \quad i = 1, 2, \dots, \lfloor N/2 \rfloor \\ p_i &= j \Omega_{Lp} \mathbf{cd}((u_i - j\nu_0) K, k), \quad i = 1, 2, \dots, \lfloor N/2 \rfloor \\ p_0 &= j \Omega_{Lp} \mathbf{sn}(j\nu_0 K, k) \end{aligned}$$

where

$$\begin{aligned} \zeta_i &= \mathbf{cd}(u_i K, k) \\ u_i &= \frac{2i-1}{N}, \quad i = 1, 2, \dots, \lfloor N/2 \rfloor \\ \nu_0 &= -\frac{j}{N K_1} \mathbf{sn}^{-1}\left(\frac{j}{\epsilon}, k_1\right) \end{aligned}$$

As N is odd, the zeros and poles occur in conjugate pairs

z_1	$j 1.2605$
z_2	$-j 1.2605$
p_0	-0.6232
p_1	$-0.1154 + j.9936$
p_2	$-0.1154 - j 0.9936$

Table 5.2: Normalized Poles and Zeros of $H_{LPF}(s_L) H_{LPF}(-s_L)$

We then have

$$H_{LPF}(s_L) = \frac{0.6236 + 0.3925 s_L^2}{0.6236 + 1.1444 s_L + 0.8539 s_L^2 + s_L^3}$$

Please see the next page

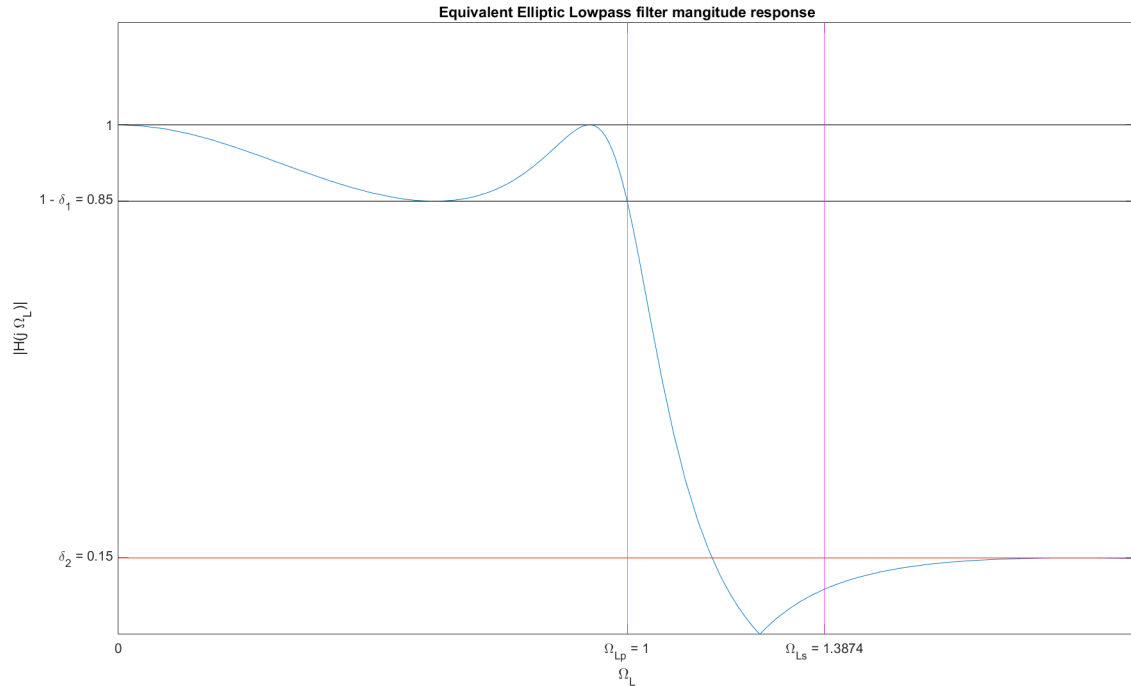


Figure 5.1: Magnitude Response of the equivalent Elliptic Lowpass Filter

5.7 Analog Bandpass Transfer Function

We now use the inverse transformation to convert the equivalent Elliptic lowpass transfer function to get the Bandpass transfer function using the relation:

$$s_L \leftarrow \frac{s^2 + \Omega_0^2}{B\Omega}$$

where B and Ω_0 have the values found in section 5.4.

$$H_{LPPF}\left(\frac{s^2 + \Omega_0^2}{B\Omega}\right) = H_{BPPF}(s)$$

$$H_{BPPF}(s) = \frac{0.1148 s^5 + 0.1342 s^3 + 0.0307 s}{s^6 + 0.2497 s^5 + 1.6483 s^4 + 0.2737 s^3 + 0.8519 s^2 + 0.0667 s + 0.1380}$$

Please see the next page

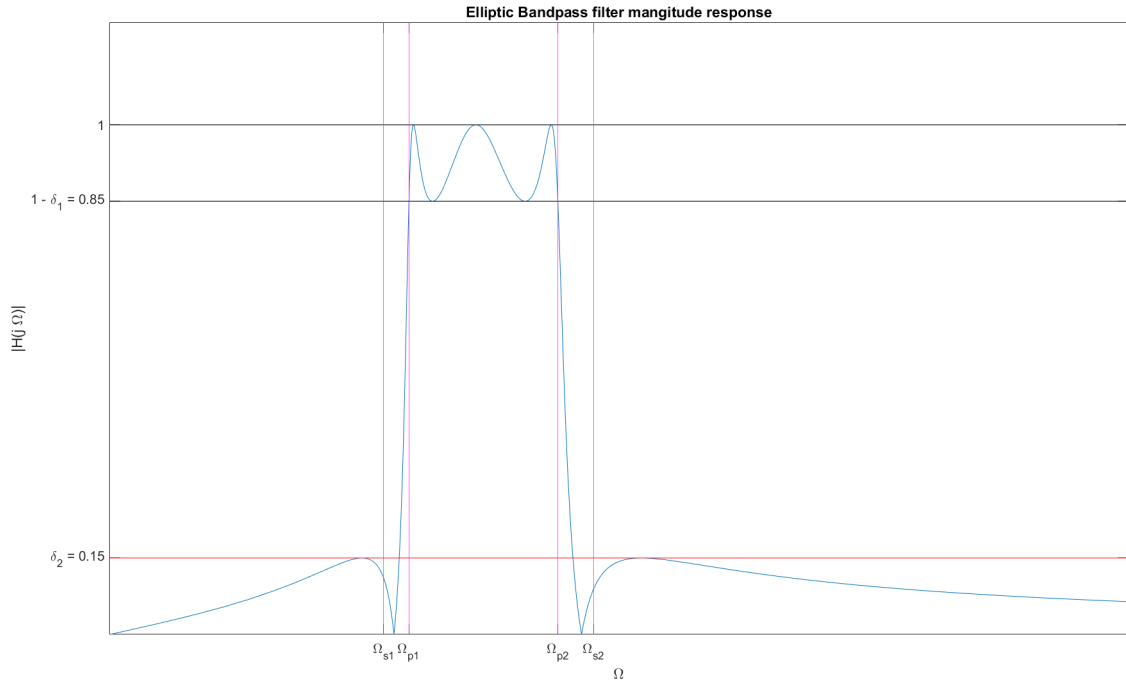


Figure 5.2: Magnitude Response of the analog BPF

5.8 Discrete-Time Bandpass Transfer Function

We now make use of the Bilinear Transformation to convert the analog bandpass filter into a discrete bandpass filter in the normalized angular frequency domain. The transform is as follows:

$$s \leftarrow \frac{1 - z^{-1}}{1 + z^{-1}}$$

The discrete time filter transfer function is

$$H_{BPF}\left(\frac{1 - z^{-1}}{1 + z^{-1}}\right) = H(z)$$

$$H(z) = \frac{0.0661 z^6 - 0.0796 z^5 + 0.0767 z^4 - 0.0767 z^2 + 0.0796 z - 0.0661}{z^6 - 1.7730 z^5 + 3.6259 z^4 - 3.3237 z^3 + 3.2660 z^2 - 1.4267 z + 0.7209}$$

Please see the next page

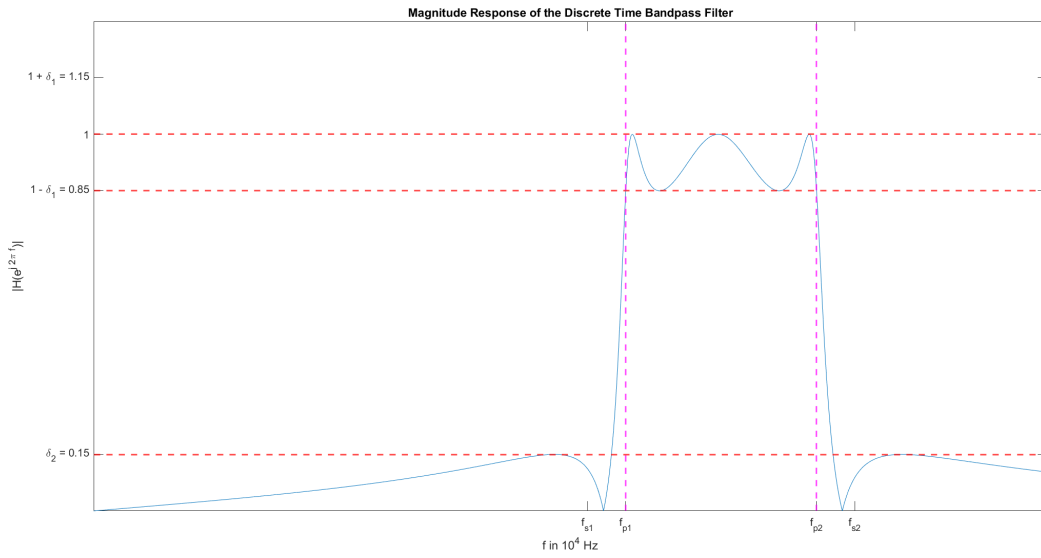


Figure 5.3: Magnitude Response of Discrete Time Bandpass Filter

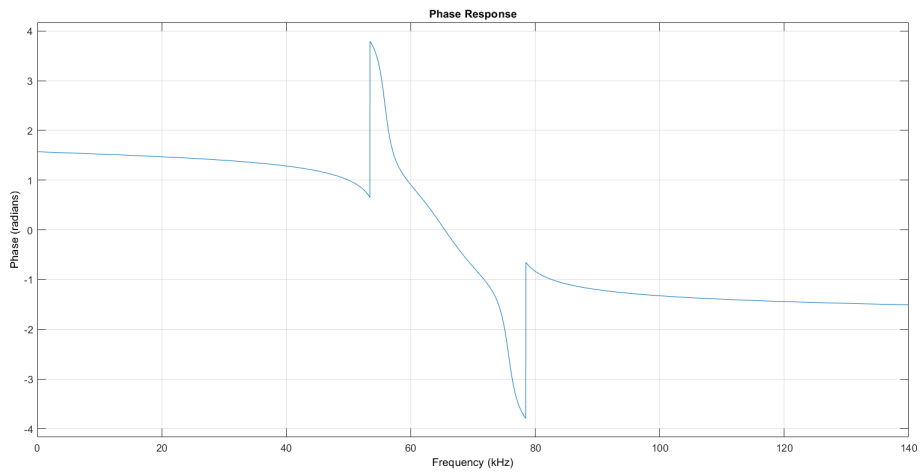


Figure 5.4: Phase Response of Discrete Time Bandpass Filter

6 Elliptic Bandstop filter design

The filter number $m = 44$. Hence,

$$q(m) = \lfloor 0.1m \rfloor = 4$$

$$r(m) = m - 10q(m) = 44 - 40 = 4$$

$$BL(m) = 25 + 1.9q(m) + 4.1r(m) = 49 \text{ kHz}$$

$$BH(m) = BL(m) + 20 = 69 \text{ kHz}$$

6.1 Unnormalized Specifications

- Lower Passband Edge = 45 kHz
- Upper passband Edge = 73 kHz
- Transition band width = 4 kHz
- Lower Stopband Edge = 49 kHz
- Upper Stopband Edge = 69 kHz
- Passband tolerance = 0.15
- Stopband tolerance = 0.15
- Passband nature = equiripple
- Stopband nature = equiripple
- Sampling frequency = 260 kHz

6.2 Normalized Specifications

The sampling angular frequency $2\pi f_s$ should map to 2π after normalization. Hence the transformation used is

$$\omega = 2\pi \frac{f_s}{f}$$

The normalized specifications are as follows (following the notations used in the lecture)

- $\omega_{p1} = 2\pi \frac{45}{260} = 0.3461 \pi$
- $\omega_{p2} = 2\pi \frac{73}{260} = 0.5615 \pi$
- $\omega_{s1} = 2\pi \frac{49}{260} = 0.3769 \pi$
- $\omega_{s2} = 2\pi \frac{69}{260} = 0.5307 \pi$
- Transition bandwidth = $2\pi \frac{4}{260} = 0.0308 \pi$
- Passband and Stopband tolerance = 0.15
- Passband nature = equiripple
- Stopband nature = equiripple

6.3 Analog Filter Specifications using Bilinear Transform

We use the following transformation to move from the z domain to the s domain.

$$s = \frac{1 - z^{-1}}{1 + z^{-1}}$$

If we put $z = j\omega$ and $s = j\Omega$, we get

$$\Omega = \tan \frac{\omega}{2}$$

The specifications now are

- Passband nature = equiripple
- Stopband nature = equiripple
- $\Omega_{s1} = 0.6725$
- $\Omega_{s2} = 1.1014$
- $\Omega_{p1} = 0.6044$
- $\Omega_{p2} = 1.2146$
- $\delta_1 = \delta_2 = 0.15$

6.4 Analog Frequency Transformation

Here we use the **Bandstop to Lowpass** analog frequency transformation, which is as follows:

$$\Omega_L = \frac{B \Omega}{\Omega_0^2 - \Omega^2}$$

where

$$\Omega_0 = \sqrt{\Omega_{p1} \Omega_{p2}} = 0.8568$$

and

$$B = \Omega_{p2} - \Omega_{p1} = 0.6102$$

Using the transformation mentioned above, we now have

- Passband and Stopband nature = monotonic
- $\Omega_{Ls1} = 1.456$
- $\Omega_{Ls2} = -1.4031$
- $\Omega_{Lp1} = +1$
- $\Omega_{Lp2} = -1$
- $\delta_1 = \delta_2 = 0.15$

6.5 Equivalent Analog Lowpass Filter Specifications

Using the more stringent specifications from those obtained in section 6.4, we have

- Passband nature = equiripple
- Stopband nature = equiripple
- $\Omega_{Ls} = \min\{|\Omega_{Ls2}|, |\Omega_{Ls1}|\} = 1.4031$
- $\Omega_{Lp} = +1$
- $\delta_1 = \delta_2 = 0.15$
- $D_1 = \frac{1}{(1-\delta_1)^2} - 1 = 0.384$
- $D_2 = \frac{1}{\delta_2^2} - 1 = 43.44$

6.6 Analog Lowpass Magnitude Response

The general transfer function for an **Elliptic** filter is given by

$$H_{LPF}(s_L) H_{LPF}(-s_L) = \frac{1}{1 + \epsilon^2 F_N^2\left(\frac{s_L}{j\Omega_{LP}}\right)}$$

with

$$F_N(w) = \text{cd}(NuK_1, k_1)$$

and

$$w = \text{cd}(uK, k)$$

where $\text{cd}(x, k)$ denotes the Jacobian elliptic function cd with modulus k and real quarter-period K .

6.6.1 Jacobian Elliptic Functions

The elliptic function $w = \text{sn}(z, k)$ is defined as follows:

$$z = \int_0^\phi \frac{d\theta}{\sqrt{1 - k^2 \sin^2 \theta}} = \int_0^w \frac{dt}{\sqrt{(1 - t^2)(1 - k^2 t^2)}}, w = \text{sn}(\phi(z, k))$$

There are three more relevant elliptic functions, cn , dn , and cd defined as:

$$\begin{aligned} w &= \text{cn}(z, k) = \cos(\phi(z, k)) \\ w &= \text{dn}(z, k) = \sqrt{1 - k^2 \text{sn}^2(z, k)} \\ w &= \text{cd}(z, k) = \frac{\text{cn}(z, k)}{\text{dn}(z, k)} \end{aligned}$$

We define the *complete elliptic integral of first kind* $K(k)$ or simply K as the value of z when $\text{sn}(z, k) = 1$ i.e. when $\phi = \pi/2$

$$K(k) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - k^2 \sin^2 \theta}}$$

Clearly $\text{cd}(K, k) = 0$. Associated with an elliptic modulus k , there is a complementary modulus $k' = \sqrt{1 - k^2}$ and its associated complete elliptic integral $K(k')$ denoted by $K'(k)$ or simply K'

$$K'(k) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - k'^2 \sin^2 \theta}} = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - (1 - k^2) \sin^2 \theta}}$$

The significance of K and K' is that sn and cd are *doubly-periodic functions* in the z plane with a real period $4K$ and a complex period $2jK'$. Some of the useful properties are as follows:

$$\text{cd}(z + (2n - 1)K, k) = (-1)^n \text{sn}(z, k) \quad (6)$$

$$\text{cd}(z + 2nK, k) = (-1)^n \text{cd}(z, k) \quad (7)$$

$$\text{cd}(z + jK', k) = \frac{1}{k \text{cd}(z, k)} \quad (8)$$

$$\text{cd}(jz, k) = \frac{1}{\text{dn}(z, k')} \quad (9)$$

$$\text{cd}(jK', k) = \frac{1}{k} \quad (10)$$

$\text{cd}(z, k)$ has zeroes at $(2m + 1)K + j2nK'$ and poles at $(2m + 1)K + j(2n + 1)K'$ for any integers m and n in the z plane.

6.6.2 Elliptic Filter Parameters

The order of the filter N is determined by putting a constraint that the magnitude response at the passband edge is not less than $1 - \delta_1$ and at the stopband edge not greater than δ_2 .

$$N = \left\lceil \frac{K K'_1}{K' K_1} \right\rceil$$

where K, K', K_1, K'_1 are defined for the complete elliptic integrals of modulus k and k_1 given by

$$k = \frac{\Omega_{Lp}}{\Omega_{Ls}} = 0.7127$$

$$k_1 = \sqrt{\frac{D_1}{D_2}} = 0.0940$$

We then have

Parameter	Value
K	1.8609
K'	1.8474
K_1	1.5743
K'_1	3.7566
N	3

Table 6.1: Elliptic Filter Parameters

We now have to find the poles of the the transfer function. The poles of the transfer function are given by the zeros of the denominator and the zeros of the transfer function are given by the poles of $F_N(\frac{s_L}{j\Omega_{Lp}})$. The zeros and poles of the transfer function are given by

$$\begin{aligned}
z_i &= j \Omega_{Lp} (k\zeta_i)^{-1}, \quad i = 1, 2, \dots, \lfloor N/2 \rfloor \\
p_i &= j \Omega_{Lp} \mathbf{cd}((u_i - j\nu_0)K, k), \quad i = 1, 2, \dots, \lfloor N/2 \rfloor \\
p_0 &= j \Omega_{Lp} \mathbf{sn}(j\nu_0K, k)
\end{aligned}$$

where

$$\begin{aligned}
\zeta_i &= \mathbf{cd}(u_iK, k) \\
u_i &= \frac{2i-1}{N}, \quad i = 1, 2, \dots, \lfloor N/2 \rfloor \\
\nu_0 &= -\frac{j}{N K_1} \mathbf{sn}^{-1}\left(\frac{j}{\epsilon}, k_1\right)
\end{aligned}$$

As N is odd, the zeros and poles occur in conjugate pairs

z_1	$j 1.2605$
z_2	$-j 1.2605$
p_0	-0.6232
p_1	$-0.1154 + j.9936$
p_2	$-0.1154 - j 0.9936$

Table 6.2: Normalized Poles and Zeros of $H_{LPF}(s_L) H_{LPF}(-s_L)$

We then have

$$H_{LPF}(s_L) = \frac{0.6236 + 0.3925 s_L^2}{0.6236 + 1.1444 s_L + 0.8539 s_L^2 + s_L^3}$$

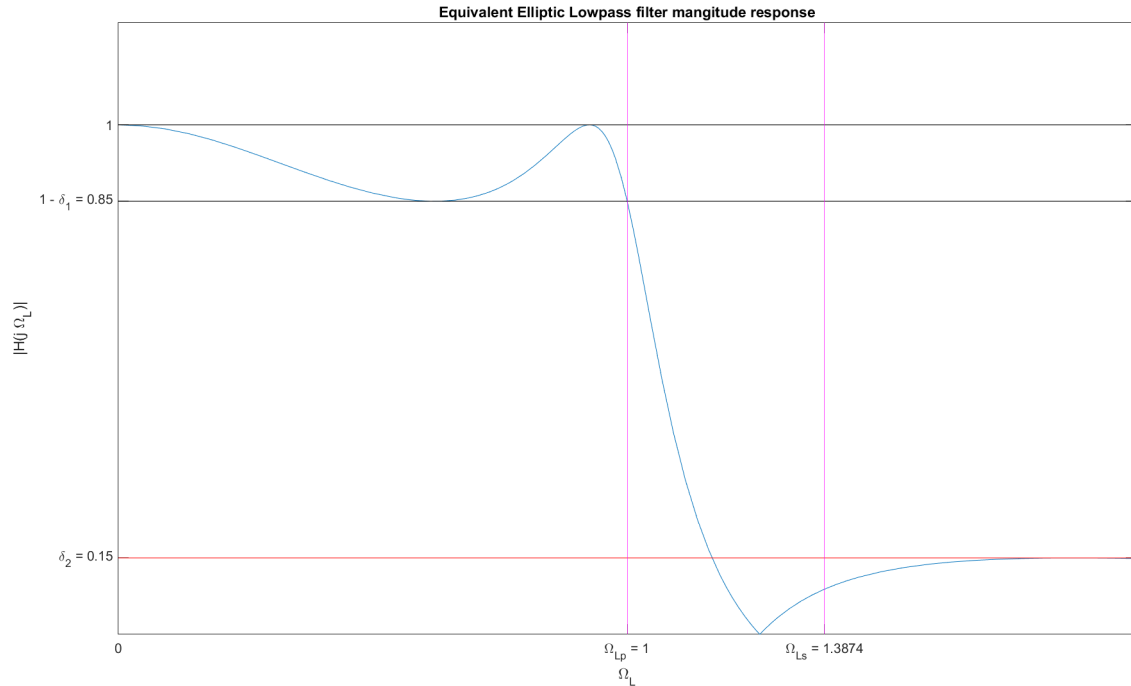


Figure 6.1: Magnitude Response of the equivalent Elliptic Lowpass Filter

6.7 Analog Bandstop Transfer Function

We now use the inverse transformation to convert the equivalent Elliptic lowpass transfer function to get the Bandstop transfer function using the relation:

$$s_L \longleftarrow \frac{B s}{s^2 + \Omega_0^2}$$

where B and Ω_0 have the values as we found in section 6.4

$$H_{LPF}\left(\frac{B s}{s^2 + \Omega_0^2}\right) = H_{BSF}(s)$$

$$H_{BSF}(s) = \frac{s^6 + 2.4367 s^4 + 1.7888 s^2 + 0.3956}{s^6 + 1.1198 s^5 + 2.7122 s^4 + 2.0085 s^3 + 1.9911 s^2 + 0.6035 s + 0.3956}$$

Please see the next page

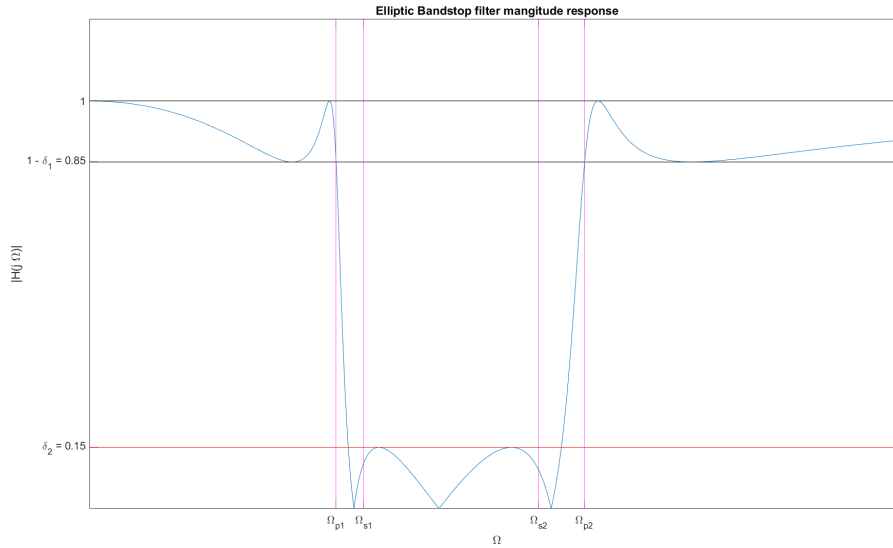


Figure 6.2: Magnitude Response of Elliptic Bandstop filter

6.8 Discrete-Time Bandstop Transfer Function

We now make use of the Bilinear Transformation to convert the analog bandstop filter into a discrete bandstop filter in the normalized angular frequency domain.

The transform is as follows:

$$s \leftarrow \frac{1 - z^{-1}}{1 + z^{-1}}$$

The discrete time filter transfer function is

$$H_{BSF}\left(\frac{1 - z^{-1}}{1 + z^{-1}}\right) = H(z)$$

$$H(z) = \frac{0.5718 z^6 - 0.5007 z^5 + 1.6997 z^4 - 0.9660 z^3 + 1.6997 z^2 - 0.5007 z + 0.5718}{z^6 - 0.7257 z^5 + 1.9146 z^4 - 0.9361 z^3 + 1.3875 z^2 - 0.3055 z + 0.2408}$$

Please see the next page

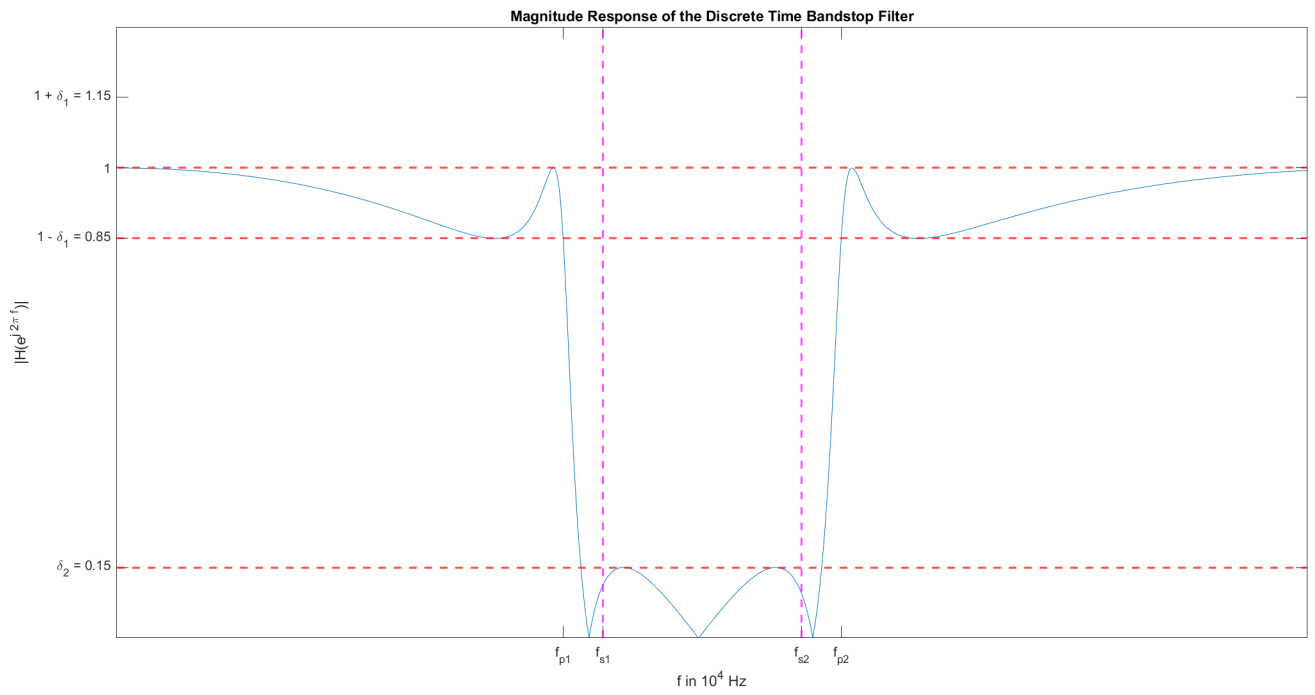


Figure 6.3: Magnitude Response of Discrete Time Bandstop Filter

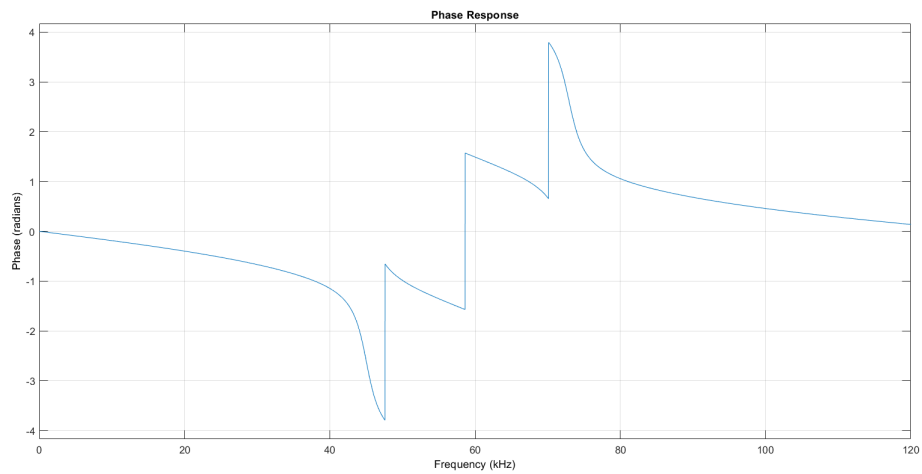


Figure 6.4: Phase Response of Discrete Time Bandstop Filter

7 Conclusion

After realising the bandpass and bandstop filters for a given set of specifications in various forms, i was able to make the following conclusions –

- Realisation using an elliptic approximation gives the least order, i.e, it requires minimum resources. The order of resources required is as follows (for a given set of specifications):

Elliptic (3) < Chebyshev (4) < Butterworth (8) < Kaiser FIR (50-60)

- The elliptic filter has the sharpest transition from passband to stopband or vice versa for the same specifications. In particular, the decreasing order of transition sharpness is

Elliptic > Chebyshev > Butterworth > Kaiser FIR

- The FIR realisation gives a linear phase response in the passband region. The nonlinearity of the phase response decreases in the following order

Elliptic > Chebyshev > Butterworth > Kaiser FIR

8 Peer Review

8.1 Review Received

Review by :[Shlok Vaibhav Singh, 18Do70064, Group 1](#)

I have reviewed this report made by Mantri Krishna Sri Ipsit, I certify that he has completed all the steps for the design of a band-pass and a band-stop filter using IIR, FIR and elliptic filter design methods.

8.2 Review Given

Review given to: [Amol Girish Shah, 18Do70005, Group 1](#)

I have reviewed the report made by Amol Girish Shah and I certified that he has completed all the steps for the design of a band-pass and a band-stop filter using IIR, FIR and elliptic filter design methods.

9 Appendix

Appendix - Contents

A	Infinite Impulse Response Bandpass Filter	50
B	Finite Impulse Response Bandpass Filter	55
C	Infinite Impulse Response Bandstop Filter	57
D	Finite Impulse Response Bandstop Filter	63
E	Elliptic Bandpass Filter	65
F	Elliptic Bandstop Filter	69
G	Helper Function for Elliptic Filter Design	73
H	References	75

A Infinite Impulse Response Bandpass Filter

```
1 clc; clear all; close all;
2 tic;
3 %% Poles of  $H_{\text{analog}}(s_L)$   $H_{\text{analog}}(-s_L)$ 
4 N = 8;
5 Omega_c = 1.0788;
6 k = 1:2*N;
7 poles = Omega_c .* exp(1i .* (pi/2) .* (1 + (2.*k + 1)./ N)
   );
8 figure();
9 plot(poles, '.', 'MarkerSize', 20);
10 xlim([-1.5 1.5]);
11 ylim([-1.5 1.5]);
12 daspect([1 1 1]);
13 hold on;
14 x = linspace(-pi, pi, 10000);
15 a = Omega_c .* cos(x);
16 b = Omega_c .* sin(x);
17 plot(a,b);
18 hold on;
19 plot(0,0,'r*');
20 left_plane_poles = [];
21 for i=1:size(poles,2)
22     hold on;
23     plot([0, real(poles(1,i))], [0, imag(poles(1,i))], 'r-')
24         );
25     if real(poles(1,i)) < 0
26         % disp(poles(1,i));
27         left_plane_poles = [left_plane_poles, poles(1,i)];
28     end
29 end
30 plot([0, 0],[-1.5, 1.5], 'k-');
31 plot([-1.5, 1.5],[0, 0], 'k-');
```

```

31 title('Poles of  $H_{\text{LPF}}(s_L)$  \,  $H_{\text{LPF}}(-s_L)$  in the  $s_L$ 
    $ plane', 'Interpreter', 'latex');
32 xlabel('\Sigma_k');
33 ylabel('\Omega_k');
34 %% Finding poles of  $H_{\text{analog}}(s_L)$ 
35 syms s x;
36 f = 1;
37 for i=1:size(left_plane_poles,2)
38     f = f * (s - left_plane_poles(1,i));
39 end
40 %% Magnitude and Phase response of  $H_{\text{analog}}(s_L)$ 
41
42 g = subs(f, s, 1i * x);
43 figure();
44 fplot(abs(1.8345/g));
45 hold on;
46 fplot(s - s - 0.15 + 1, 'k-', 'Markersize', 10);
47 hold on;
48 fplot(x-x+1, 'k-', 'Markersize', 10);
49 hold on;
50 fplot(x - x + 0.15, 'r-', 'Markersize', 10);
51 xline(1, 'magenta-');
52 hold on;
53 xline(1.3874, 'magenta-');
54 axis([0 2 0 1.2]);
55 set(gca, 'XTick', [0, 1, 1.3874], 'xticklabel', {'0', '\Omega_{Lp} = 1', '\Omega_{Ls} = 1.3874'});
56 set(gca, 'YTick', [0.15, 0.85, 1], 'yticklabel', {'\delta_2 = 0.15', '1 - \delta_1 = 0.85', '1'});
57 daspect([1 1 1]);
58 title('Equivalent Butterworth Lowpass filter mangitude
    response');
59 xlabel('\Omega_L');

```

```

60 ylabel(' |H(j \Omega_L)| ');
61
62 figure();
63 fplot(angle(1.8345/g));
64 xlabel('\Omega_L');
65 ylabel('\angle H(j \Omega_L)');
66 title('Equivalent Butterworth Lowpass filter phase response
        ');
67 %% Bandpass analog frequency transformation
68 syms s1;
69 Omega_o = 0.7189;
70 B = 0.2924;
71 s1 = (s^2 + Omega_o^2) ./ (B * s);
72 h = subs(f, s, s1);
73
74 %disp(1.8345/h); % transformed i.e., H_{analog}(s)
75
76 a = vpa(1.8345/h, 5); % bandpass system function
77 %%
78 b = subs(a, s, 1i * x); % to get the frequency response
79 figure();
80 fplot(abs(b));
81 set(gca, 'XTick', [0.5375, 0.5875, 0.8799, 0.9498], '
        xticklabel', {'\Omega_{s1}', '\Omega_{p1}', '\Omega_{p2}'
        ', '\Omega_{s2}'});
82 set(gca, 'YTick', [0.15, 0.85, 1], 'yticklabel', {'\delta_2
        = 0.15', '1 - \delta_1 = 0.85', '1'});
83 hold on;
84 xline(0.5375, 'magenta-');hold on;
85 xline(0.5875, 'magenta-');hold on;
86 xline(0.8799, 'magenta-');hold on;
87 xline( 0.9498, 'magenta-');
88 hold on;

```



```

89 fplot(s - s - 0.15 + 1, 'k-', 'Markersize', 10);
90 hold on;
91 fplot(x-x+1, 'k-', 'Markersize', 10);
92 hold on;
93 fplot(x - x + 0.15, 'r-', 'Markersize', 10);
94 daspect([1 1 1]);
95 title('Butterworth Bandpass filter mangitude response');
96 xlabel('\Omega');
97 ylabel('|H(j \Omega)|');
98 axis([0 2 0 1.2]);
99
100 figure();
101 fplot(angle(b));
102 daspect([1 1 1]);
103 title('Butterworth Bandpass filter phase response');
104 xlabel('\Omega');
105 ylabel('\angle H(j \Omega)');
106 %% bilinear transformation
107 syms w;
108 c = subs(a, s, (1-1/w) / (1+1/w));
109 [Nz, Dz] = numden(1/c);
110 Nz = sym2poly(Nz);
111 Dz = sym2poly(Dz);
112 % disp('#####');
113 % disp(Nz);
114 % disp(Dz);
115 %%
116 [H, f] = freqz(Dz, Nz, 1024*1024, 330e3);
117 plot(f, abs(H));
118 axis([0 100e3 0 1.3]);
119 hold on;
120 yline(1.15, 'red--', 'LineWidth', 1.5);
121 hold on;

```

```

122 yline(0.85, 'red--', 'LineWidth', 1.5);
123 hold on;
124 yline(0.15, 'red--', 'LineWidth', 1.5);
125 hold on;
126 xline(55.8e3, 'magenta--', 'LineWidth', 1.5);
127 hold on;
128 xline(75.8e3, 'magenta--', 'LineWidth', 1.5);
129 xlabel('f in 10^4 Hz');
130 ylabel('|H(e^{j 2\pi f})|');
131 title('Magnitude Response of the Discrete Time Bandpass
        Filter');
132 set(gca, 'XTick', [51.8e3, 55.8e3, 75.8e3, 79.8e3], '
        xticklabel', {'f_{s1}', 'f_{p1}', 'f_{p2}', 'f_{s2}'});
133 set(gca, 'YTick', [0.15, 0.85, 1, 1.15], 'yticklabel', {'\
        delta_2 = 0.15', '1 - \delta_1 = 0.85', '1', '1 + \
        delta_1 = 1.15'});
134 %%
135 figure();
136 plot(f, angle(H));
137 % set(gca, 'XTick', [51.8e3, 55.8e3, 75.8e3, 79.8e3], '
        xticklabel', {'f_{s1}', 'f_{p1}', 'f_{p2}', 'f_{s2}'});
138 xlim([1e4 120e3]);
139 ylabel('\angle H(e^{j 2\pi f})');
140 xlabel('f in Hz');
141 title('Phase Response of the Discrete Time Bandpass Filter '
        );

```

B Finite Impulse Response Bandpass Filter

```
1 %%
2 clc; clear all; close all;
3 %% BP Filter Specs
4 omega_s1 = 0.314 * pi;
5 omega_s2 = 0.4836 * pi;
6 omega_p1 = 0.3382 * pi;
7 omega_p2 = 0.4594 * pi;
8 transition_bw = 0.0242 * pi;
9 delta = 0.15;
10 omega_c1 = 0.5 * (omega_p1 + omega_s1);
11 omega_c2 = 0.5 * (omega_p2 + omega_s2);
12 %% Kaiser window parameters
13 A = -20 * log10(delta);
14 fprintf('A = %f\n', A);
15
16 width_min = ceil( 1 + ((A - 8)/(2.285 * transition_bw)));
17 fprintf('Minimum width = %d\n', width_min);
18
19 alpha = -1;
20 if A < 21
21     alpha = 0;
22 elseif A >=21 && A <= 50
23     alpha = 0.5842 * (A - 21) ^ 0.4 + 0.07886 * (A - 21);
24 elseif A > 50
25     alpha = 0.1102 * (A - 8.7);
26 else
27
28 end
29 beta = alpha /width_min;
30 fprintf('beta = %f\n', beta);
31 %%
32 width = width_min + 16;
```

```

33 w = kaiser(width, beta);
34 ideal_bpf = ideal_lpf(omega_c2, width) - ideal_lpf(omega_c1
    , width);
35 fir_bpf = ideal_bpf .* w';
36 [H, f] = freqz(fir_bpf, 1, 1024, 330e3);
37 figure();
38 plot(f, abs(H));
39 hold on;
40 xline(55.8e3, 'magenta--', 'LineWidth', 1.5);
41 hold on;
42 xline(75.8e3, 'magenta--', 'LineWidth', 1.5);
43 hold on;
44 yline(1.15, 'red--', 'LineWidth', 1.5);
45 hold on;
46 yline(0.85, 'red--', 'LineWidth', 1.5);
47 hold on;
48 yline(0.15, 'red--', 'LineWidth', 1.5);
49 xlabel('f in 10^4 Hz');
50 ylabel('|H(e^{j2 \pi f})|');
51 title('Magnitude Response of Discrete Time FIR Bandpass
    Filter');
52 set(gca, 'XTick', [51.8e3, 55.8e3, 75.8e3, 79.8e3], '
    xticklabel', {'f_{s1}', 'f_{p1}', 'f_{p2}', 'f_{s2}'});
53 set(gca, 'YTick', [0.15, 0.85, 1, 1.15], 'yticklabel', {'\
    delta_2 = 0.15', '1 - \delta_1 = 0.85', '1', '1 + \
    delta_1 = 1.15'});
54
55 %%
56 disp(fir_bpf);
57 %%
58 fvtool(fir_bpf, 'Analysis', 'Phase');
59 fvtool(fir_bpf, 'Analysis', 'Impulse');

```

C Infinite Impulse Response Bandstop Filter

```
1 clc; clear all; close all;
2 % transformed normalized filter specs
3 syms x B Omega_o f(x);
4 B = 0.6102;
5 Omega_o = 0.8568;
6 f(x) = (B * x) / (-x^2 + Omega_o^2);
7 disp(vpa(f(0.6725), 5));
8 disp(vpa(f(1.1014), 5));
9 disp(vpa(f(0.6044), 5));
10 disp(vpa(f(1.2146), 5));
11 %% equivaent lowpass filter specs
12 D2 = 43.44;
13 D1 = 0.384;
14 Omega_ls = 1.4031;
15 Omega_lp = 1;
16 epsilon = sqrt(D1);
17 N = ceil((acosh(sqrt(D2/D1))) / (acosh(Omega_ls / Omega_lp)
18     ));
19 fprintf('N = %d\n', N);
20 %% finding out the left half poles
21 k = 0:2 * N - 1;
22 Ak = (2 * k + 1) .* (pi / (2 * N));
23 Bk = (1 / N) * asinh(1 / epsilon);
24 poles = (-1 * sin(Ak) * sinh(Bk)) + 1i * (cos(Ak) * cosh(Bk
25     ));
26 figure();
27 plot(poles, '.', 'MarkerSize', 20);
28 daspect([1 1 1]);
29 hold on;
30 x = linspace(-pi, pi, 10000);
31 a = sinh(Bk) .* cos(x);
32 b = cosh(Bk) .* sin(x);
```

```

31 plot(a,b);
32 hold on;
33 plot(o,o,'r*');
34 xlim([-1.5 1.5]);
35 ylim([-1.5 1.5]);
36 left_plane_poles = [];
37 for i=1:size(poles,2)
38     hold on;
39     plot([o, real(poles(1,i))], [o, imag(poles(1,i))], 'r-'
40         );
41     if real(poles(1,i)) < o
42         disp(poles(1,i));
43         left_plane_poles = [left_plane_poles , poles(1,i)];
44     end
45 end
46 plot([o, o],[-1.5, 1.5], 'k-');
47 plot([-1.5, 1.5],[o, o], 'k-');
48 title('Poles of  $H_{\text{LPF}}(s_L)$  \,  $H_{\text{LPF}}(-s_L)$  in the  $s_L$ 
49     $ plane', 'Interpreter', 'latex');
50 xlabel('\Sigma_k');
51 ylabel('\Omega_k');
52 %% Analog Lowpass transfer function in s_L domain
53 num = 1;
54 den = 1;
55 syms sl;
56 for i=1:size(left_plane_poles, 2)
57     num = num * left_plane_poles(1,i);
58     den = den * (sl - left_plane_poles(1,i));
59 end
60 H_LPF = num / (den * sqrt(1 + D1));
61 disp(expand(vpa(den * sqrt(1 + D1), 5)));
62 disp(num);
63 syms Omega_L;

```

```

62 H_LPF_freq = subs(H_LPF, s1, 1i * Omega_L);
63 figure();
64 fplot(abs(H_LPF_freq));
65 hold on;
66 fplot(s1 - s1 - 0.15 + 1, 'k-', 'Markersize', 10);
67 hold on;
68 fplot(s1-s1+1, 'k-', 'Markersize', 10);
69 hold on;
70 fplot(s1 - s1 + 0.15, 'r-', 'Markersize', 10);
71 xline(1, 'magenta-');
72 hold on;
73 xline(1.4031, 'magenta-');
74 axis([0 2 0 1.2]);
75 set(gca, 'XTick', [0, 1, 1.4031], 'xticklabel', {'0', '\
      Omega_{Lp} = 1', '\Omega_{Ls} = 1.4031'});
76 set(gca, 'YTick', [0.15, 0.85, 1], 'yticklabel', {'\delta_2
      = 0.15', '1 - \delta_1 = 0.85', '1'});
77 daspect([1 1 1]);
78 title('Equivalent Chebyshev Lowpass filter magnitude
      response');
79 xlabel('\Omega_L');
80 ylabel('|H(j \Omega_L)|');
81
82 figure();
83 fplot(angle(H_LPF_freq));
84 xlabel('\Omega_L');
85 ylabel('\angle H(j \Omega_L)');
86 title('Equivalent Chebyshev Lowpass filter phase response')
      ;
87 %% Chebyshev lowpass magnitude response
88 syms s;
89 H_BSF = subs(H_LPF, s1, (B * s)/(s^2 + Omega_o^2));
90 [N, D] = numden(H_BSF);

```

```

91 disp(expand(vpa(N, 2)));
92 disp(expand(vpa(D, 2)));
93 %% Chebyshev bandstop magnitude response
94 syms Omega;
95 N1 = subs(N* 1e-138, s, 1i * Omega);
96 D1 = subs(D * 1e-138, s, 1i * Omega);
97 H_BSF_freq_resp = (N1) / (D1);
98 H_BSF_freq_resp = vpa(H_BSF_freq_resp, 2);
99
100 figure();
101 fplot(abs(H_BSF_freq_resp));
102 set(gca, 'XTick', [0.6044, 0.6725, 1.1014, 1.2146], '
    xticklabel', {'\Omega_{p1}', '\Omega_{s1}', '\Omega_{s2}'
    ', '\Omega_{p2}'});
103 set(gca, 'YTick', [0.15, 0.85, 1], 'yticklabel', {'\delta_2
    = 0.15', '1 - \delta_1 = 0.85', '1'});
104 hold on;
105 xline(0.6044, 'magenta-');hold on;
106 xline(0.6725, 'magenta-');hold on;
107 xline(1.1014, 'magenta-');hold on;
108 xline(1.2146, 'magenta-');
109 hold on;
110 fplot(s - s - 0.15 + 1, 'k-', 'Markersize', 10);
111 hold on;
112 fplot(s-s+1, 'k-', 'Markersize', 10);
113 hold on;
114 fplot(s -s + 0.15, 'r-', 'Markersize', 10);
115 daspect([1 1 1]);
116 title('Chebyshev Bandstop filter mangitude response');
117 xlabel('\Omega');
118 ylabel('|H(j \Omega)|');
119 axis([0 2 0 1.2]);
120 %% Chebyshev bandstop phase response

```



```

121 figure ();
122 fplot (angle (H_BSF_freq_resp));
123 daspect ([1 1 1]);
124 title ('Chebyshev Bandstop filter phase response');
125 xlabel ('\Omega');
126 ylabel ('\angle H(j \Omega)');
127 %% Bilinear transformation from analog to digital
128 syms z;
129 Hz = subs (H_BSF, s, (z - 1) / (z + 1));
130 [Nz, Dz] = numden (Hz);
131 disp (expand (vpa (Nz, 2)));
132 disp (expand (vpa (Dz, 2)));
133 Nzz = sym2poly (Nz);
134 Dzz = sym2poly (Dz);
135 [H, f] = freqz (Nzz, Dzz, 1024*1024, 260e3);
136 plot (f, abs (H));
137 axis ([0 100e3 0 1.3]);
138 hold on;
139 yline (1.15, 'red--', 'LineWidth', 1.5);
140 hold on;
141 yline (0.85, 'red--', 'LineWidth', 1.5);
142 hold on;
143 yline (0.15, 'red--', 'LineWidth', 1.5);
144 hold on;
145 xline (49e3, 'magenta--', 'LineWidth', 1.5);
146 hold on;
147 xline (69e3, 'magenta--', 'LineWidth', 1.5);
148 xlabel ('f in 10^4 Hz');
149 ylabel ('|H(e^{j 2\pi f})|');
150 title ('Magnitude Response of the Discrete Time Bandstop
        Filter');
151 set (gca, 'XTick', [45e3, 49e3, 69e3, 73e3], 'xticklabel', {
        'f_{p1}', 'f_{s1}', 'f_{s2}', 'f_{p2}}');

```

```

152 set(gca, 'YTick', [0.15, 0.85, 1, 1.15], 'yticklabel', {'\
    delta_2 = 0.15', '1 - \delta_1 = 0.85', '1', '1 + \
    delta_1 = 1.15'});
153 %% Phase response of DT BSF
154 figure();
155 plot(f, angle(H));
156 xlim([1e4 110e3]);
157 ylabel('\angle H(e^{j 2\pi f})');
158 xlabel('f in Hz');
159 title('Phase Response of the Discrete Time Bandstop Filter '
    );
160 hold on;
161 xline(49e3, 'magenta--', 'LineWidth', 1);
162 hold on;
163 xline(69e3, 'magenta--', 'LineWidth', 1);
164 set(gca, 'XTick', [1e4, 45e3, 49e3, 69e3, 73e3, 110e3], '
    xticklabel', {1e4, 'f_{p1}', 'f_{s1}', 'f_{s2}', 'f_{p2}
    ', 110e3});
165 %%
166 % fvtool(Nzz, Dzz);

```

D Finite Impulse Response Bandstop Filter

```
1 %%
2 clc; clear all; close all;
3 %% BS Filter Specs
4 omega_s1 = 0.3769 * pi;
5 omega_s2 = 0.5307 * pi;
6 omega_p1 = 0.3461 * pi;
7 omega_p2 = 0.5615 * pi;
8 transition_bw = 0.0308 * pi;
9 delta = 0.15;
10 omega_c1 = 0.5 * (omega_p1 + omega_s1);
11 omega_c2 = 0.5 * (omega_p2 + omega_s2);
12 %% Kaiser window parameters
13 A = -20 * log10(delta);
14 fprintf('A = %f\n', A);
15
16 width_min = ceil( 1 + ((A - 8)/(2.285 * transition_bw)));
17 fprintf('Minimum width = %d\n', width_min);
18
19 alpha = -1;
20 if A < 21
21     alpha = 0;
22 elseif A >=21 && A <= 50
23     alpha = 0.5842 * (A - 21) ^ 0.4 + 0.07886 * (A - 21);
24 elseif A > 50
25     alpha = 0.1102 * (A - 8.7);
26 else
27
28 end
29 beta = alpha /width_min;
30 fprintf('beta = %f\n', beta);
31 %% Magnitude Response
32 width = width_min + 13;
```

```

33 w = kaiser(width, beta);
34 ideal_bsf = ideal_lpf(pi, width) - ideal_lpf(omega_c2,
        width) + ideal_lpf(omega_c1, width);
35 fir_bsf = ideal_bsf .* w';
36 [H, f] = freqz(fir_bsf, 1, 1024, 260e3);
37 figure();
38 plot(f, abs(H));
39 hold on;
40 xline(49e3, 'magenta--', 'LineWidth', 1.5);
41 hold on;
42 xline(69e3, 'magenta--', 'LineWidth', 1.5);
43 hold on;
44 yline(1.15, 'red--', 'LineWidth', 1.5);
45 hold on;
46 yline(0.85, 'red--', 'LineWidth', 1.5);
47 hold on;
48 yline(0.15, 'red--', 'LineWidth', 1.5);
49 xlabel('f in 10^4 Hz');
50 ylabel('|H(e^{j2 \pi f})|');
51 title('Magnitude Response of Discrete Time FIR Bandstop
        Filter');
52 set(gca, 'XTick', [45e3, 49e3, 69e3, 73e3], 'xticklabel', {
        'f_{p1}', 'f_{s1}', 'f_{s2}', 'f_{p2}}');
53 set(gca, 'YTick', [0.15, 0.85, 1, 1.15], 'yticklabel', {'\
        delta_2 = 0.15', '1 - \delta_1 = 0.85', '1', '1 + \
        delta_1 = 1.15'});
54 %%
55 disp(fir_bsf);
56 %%
57 fvtool(fir_bsf, 'Analysis', 'Phase');
58 fvtool(fir_bsf, 'Analysis', 'Impulse');

```

E Elliptic Bandpass Filter

```
1 % Please execute this code section wise
2 % so that the values of the variables
3 % settle and are stable
4 %
5
6 clc; clear ; close all;
7 %% Elliptic Lowpass filter transfer function and plots
8 [z, p, Ho, B, A] = ellipap2(3, 1.411, 16.48);
9 syms sl Omega_L;
10 H_LPF = (1 + 0.6294 * sl^2) / ((1 + 1.6046 * sl) * (1 +
    0.2306 * sl + 0.9994 * sl^2));
11 H_LPF_freq = subs(H_LPF, sl, 1i * Omega_L);
12 [ns, ds] = numden(H_LPF);
13 nsl = sym2poly(ns);
14 dsl = sym2poly(ds);
15 k = ds(1);
16 nsl = nsl / k;
17 ds = ds / k;
18 disp(nsl);
19 disp(dsl);
20 figure();
21 fplot(abs(H_LPF_freq));
22 hold on;
23 fplot(sl - sl - 0.15 + 1, 'k-', 'Markersize', 10);
24 hold on;
25 fplot(sl - sl + 1, 'k-', 'Markersize', 10);
26 hold on;
27 fplot(sl - sl + 0.15, 'r-', 'Markersize', 10);
28 xline(1, 'magenta-');
29 hold on;
30 xline(1.3874, 'magenta-');
31 axis([0 2 0 1.2]);
```

```

32 set(gca, 'XTick', [0, 1, 1.3874], 'xticklabel', {'0', '\
    Omega_{Lp} = 1', '\Omega_{Ls} = 1.3874'});
33 set(gca, 'YTick', [0.15, 0.85, 1], 'yticklabel', {'\delta_2
    = 0.15', '1 - \delta_1 = 0.85', '1'});
34 daspect([1 1 1]);
35 title('Equivalent Elliptic Lowpass filter mangitude
    response');
36 xlabel('\Omega_L');
37 ylabel('|H(j \Omega_L)|');
38
39 %% Analog lowpass to bandpass frequency transformation
40 syms s Omega;
41 Omega_o = 0.7189;
42 B = 0.2924;
43 H_BPF = subs(H_LPF, s1, (s^2 + Omega_o^2) / (B * s));
44 [ns, ds] = numden(H_BPF);
45 ns = sym2poly(ns);
46 ds = sym2poly(ds);
47 k = ds(1);
48 ns = ns / k;
49 ds = ds / k;
50 disp(ns);
51 disp(ds);
52 H_BPF_freq = subs(H_BPF, s, 1i * Omega);
53 figure();
54 fplot(abs(H_BPF_freq));
55 set(gca, 'XTick', [0.5375, 0.5875, 0.8799, 0.9498], '
    xticklabel', {'\Omega_{s1}', '\Omega_{p1}', '\Omega_{p2}
    ', '\Omega_{s2}'});
56 set(gca, 'YTick', [0.15, 0.85, 1], 'yticklabel', {'\delta_2
    = 0.15', '1 - \delta_1 = 0.85', '1'});
57 hold on;
58 xline(0.5375, 'magenta-');hold on;

```

```

59 xline(0.5875, 'magenta-');hold on;
60 xline(0.8799, 'magenta-');hold on;
61 xline( 0.9498, 'magenta-');
62 hold on;
63 fplot(s - s - 0.15 + 1, 'k-', 'Markersize', 10);
64 hold on;
65 fplot(s-s+1, 'k-', 'Markersize', 10);
66 hold on;
67 fplot(s - s + 0.15, 'r-', 'Markersize', 10);
68 daspect([1 1 1]);
69 title('Elliptic Bandpass filter mangitude response');
70 xlabel('\Omega');
71 ylabel('|H(j \Omega)|');
72 axis([0 2 0 1.2]);
73 %% Analog to z bilinear transformation
74 syms z;
75 Hz = subs(H_BPF, s, (z-1)/(z+1));
76 [Nz, Dz] = numden(Hz);
77 Nz = sym2poly(Nz);
78 Dz = sym2poly(Dz);
79 k = Dz(1);
80 Nz = Nz / k;
81 Dz = Dz / k;
82 disp(Nz);
83 disp(Dz);
84 [H, f] = freqz(Nz,Dz,1024*1024, 330e3);
85 plot(f, abs(H));
86 axis([0 100e3 0 1.3]);
87 hold on;
88 yline(1.00, 'red--', 'LineWidth', 1.5);
89 hold on;
90 yline(0.85, 'red--', 'LineWidth', 1.5);
91 hold on;

```

```

92 yline(0.15, 'red--', 'LineWidth', 1.5);
93 hold on;
94 xline(55.8e3, 'magenta--', 'LineWidth', 1.5);
95 hold on;
96 xline(75.8e3, 'magenta--', 'LineWidth', 1.5);
97 xlabel('f in 10^4 Hz');
98 ylabel('|H(e^{j 2\pi f})|');
99 title('Magnitude Response of the Discrete Time Bandpass
      Filter');
100 set(gca, 'XTick', [51.8e3, 55.8e3, 75.8e3, 79.8e3], '
      xticklabel', {'f_{s1}', 'f_{p1}', 'f_{p2}', 'f_{s2}'});
101 set(gca, 'YTick', [0.15, 0.85, 1, 1.15], 'yticklabel', {'\
      delta_2 = 0.15', '1 - \delta_1 = 0.85', '1', '1 + \
      delta_1 = 1.15'});
102 fvtool(Nz, Dz, 'Analysis', 'Phase');

```


F Elliptic Bandstop Filter

```
1 % Please execute this code section wise
2 % so that the values of the variables
3 % settle and are stable
4 %
5
6 clc; clear ; close all;
7 %
8 %% Elliptic Lowpass filter transfer function and plots
9 [z, p, Ho, B, A] = ellipap2(3, 1.411, 16.48);
10 syms sl Omega_L;
11 H_LPF = (1 + 0.6294 * sl^2) / ((1 + 1.6046 * sl) * (1 +
    0.2306 * sl + 0.9994 * sl^2));
12 H_LPF_freq = subs(H_LPF, sl, 1i * Omega_L);
13 [ns, ds] = numden(H_LPF);
14 nsl = sym2poly(ns);
15 dsl = sym2poly(ds);
16 k = ds(1);
17 nsl = nsl / k;
18 ds = ds / k;
19 disp(nsl);
20 disp(dsl);
21 figure();
22 fplot(abs(H_LPF_freq));
23 hold on;
24 fplot(sl - sl - 0.15 + 1, 'k-', 'Markersize', 10);
25 hold on;
26 fplot(sl - sl + 1, 'k-', 'Markersize', 10);
27 hold on;
28 fplot(sl - sl + 0.15, 'r-', 'Markersize', 10);
29 xline(1, 'magenta-');
30 hold on;
31 xline(1.3874, 'magenta-');
```

```

32 axis([0 2 0 1.2]);
33 set(gca, 'XTick', [0, 1, 1.3874], 'xticklabel', {'0', '\
    Omega_{Lp} = 1', '\Omega_{Ls} = 1.3874'});
34 set(gca, 'YTick', [0.15, 0.85, 1], 'yticklabel', {'\delta_2
    = 0.15', '1 - \delta_1 = 0.85', '1'});
35 daspect([1 1 1]);
36 title('Equivalent Elliptic Lowpass filter mangitude
    response');
37 xlabel('\Omega_L');
38 ylabel('|H(j \Omega_L)|');
39 % figure();
40 % fplot(angle(H_LPF_freq));
41 % xlabel('\Omega_L');
42 % ylabel('\angle H(j \Omega_L)');
43 % title('Equivalent Elliptic Lowpass filter phase response
    ');
44 %% Analog lowpass to bandstop frequency transformation
45 syms s Omega;
46 B = 0.6102;
47 Omega_o = 0.8568;
48 H_BSF = subs(H_LPF, s1, (B * s) / (s^2 + Omega_o^2));
49 [ns, ds] = numden(H_BSF);
50 ns = sym2poly(ns);
51 ds = sym2poly(ds);
52 k = ds(1);
53 ns = ns / k;
54 ds = ds / k;
55 disp(ns);
56 disp(ds);
57 H_BSF_freq = subs(H_BSF, s, 1i * Omega);
58 figure();
59 fplot(abs(H_BSF_freq));

```

```

60 set(gca, 'XTick', [0.6044, 0.6725, 1.1014, 1.2146], '
    xticklabel', {'\Omega_{p1}', '\Omega_{s1}', '\Omega_{s2}'
    ', '\Omega_{p2}'});
61 set(gca, 'YTick', [0.15, 0.85, 1], 'yticklabel', {'\delta_2
    = 0.15', '1 - \delta_1 = 0.85', '1'});
62 hold on;
63 xline(0.6044, 'magenta-');hold on;
64 xline(0.6725, 'magenta-');hold on;
65 xline(1.1014, 'magenta-');hold on;
66 xline(1.2146, 'magenta-');
67 hold on;
68 fplot(s - s - 0.15 + 1, 'k-', 'Markersize', 10);
69 hold on;
70 fplot(s-s+1, 'k-', 'Markersize', 10);
71 hold on;
72 fplot(s -s + 0.15, 'r-', 'Markersize', 10);
73 daspect([1 1 1]);
74 title('Elliptic Bandstop filter mangitude response');
75 xlabel('\Omega');
76 ylabel('|H(j \Omega)|');
77 axis([0 2 0 1.2]);
78 %% Analog to z bilinear transformation
79 syms z;
80 Hz = subs(H_BSF, s, (z-1)/(z+1));
81 [Nz, Dz] = numden(Hz);
82 Nz = sym2poly(Nz);
83 Dz = sym2poly(Dz);
84 k = Dz(1);
85 Nz = Nz / k;
86 Dz = Dz / k;
87 disp(Nz);
88 disp(Dz);
89 [H, f] = freqz(Nz,Dz,1024*1024, 260e3);

```

```

90 plot(f,abs(H));
91 axis([0 120e3 0 1.3]);
92 hold on;
93 yline(1, 'red--', 'LineWidth', 1.5);
94 hold on;
95 yline(0.85, 'red--', 'LineWidth', 1.5);
96 hold on;
97 yline(0.15, 'red--', 'LineWidth', 1.5);
98 hold on;
99 xline(49e3, 'magenta--', 'LineWidth', 1.5);
100 hold on;
101 xline(69e3, 'magenta--', 'LineWidth', 1.5);
102 xlabel('f in 10^4 Hz');
103 ylabel('|H(e^{j 2\pi f})|');
104 title('Magnitude Response of the Discrete Time Bandstop
        Filter');
105 set(gca, 'XTick', [45e3, 49e3, 69e3, 73e3], 'xticklabel', {
        'f_{p1}', 'f_{s1}', 'f_{s2}', 'f_{p2}}');
106 set(gca, 'YTick', [0.15, 0.85, 1, 1.15], 'yticklabel', {'\
        delta_2 = 0.15', '1 - \delta_1 = 0.85', '1', '1 + \
        delta_1 = 1.15'});
107 fvtool(Nz, Dz, 'Analysis', 'Phase');

```

G Helper Function for Elliptic Filter Design

```
1 function [z,p,Ho,B,A] = ellipap2(N,Ap,As)
2 if nargin==0, help ellipap2; return; end
3 Gp = 10^(-Ap/20); % passband gain
4 ep = sqrt(10^(Ap/10) - 1); % ripple factors
5 es = sqrt(10^(As/10) - 1);
6 k1 = ep/es;
7 k = ellipdeg(N,k1); % solve degree equation
8 L = floor(N/2); r = mod(N,2); % L is the number of second-
   order sections
9 i = (1:L)'; ui = (2*i-1)/N; zeta_i = cde(ui,k); % zeros of
   elliptic rational function
10 z = j./(k*zeta_i); % filter zeros = poles of elliptic
   rational function
11 vo = -j*asne(j/ep, k1)/N; % solution of sn(jv0NK1, k1) = j
   /?p
12 p = j*cde(ui-j*vo, k); % filter poles
13 po = j*sne(j*vo, k); % first-order pole, needed when N is
   odd
14 B = [ones(L,1), -2*real(1./z), abs(1./z).^2]; % second-
   order numerator sections
15 A = [ones(L,1), -2*real(1./p), abs(1./p).^2]; % second-
   order denominator sections
16 if r==0, % prepend first-order sections
17 B = [Gp, 0, 0; B]; A = [1, 0, 0; A];
18 else
19 B = [1, 0, 0; B]; A = [1, -real(1/po), 0; A];
20 end
21 z = cplxpair([z; conj(z)]); % append conjugate zeros
22 p = cplxpair([p; conj(p)]); % append conjugate poles
23 if r==1, p = [p; po]; end % append first-order pole when N
   is odd
24 Ho = Gp^(1-r); % dc gain
```

25 | end

H References

1. Prof. V.M.Gadre's Video Lectures and Slides
2. [Lecture Notes on Elliptic Filter Design by Sophocles J. Orfanidis](#)

The helper function for the elliptic filter design is taken from this source.